Moduli Spaces in Algebraic Geometry

Math 245 A (winter 2022)

Feb. 25, 2022.

Moduli Space down-to-earth.

Spec $\mathbb{Z}$.

singular

Mnëv's Universality Theorem '86

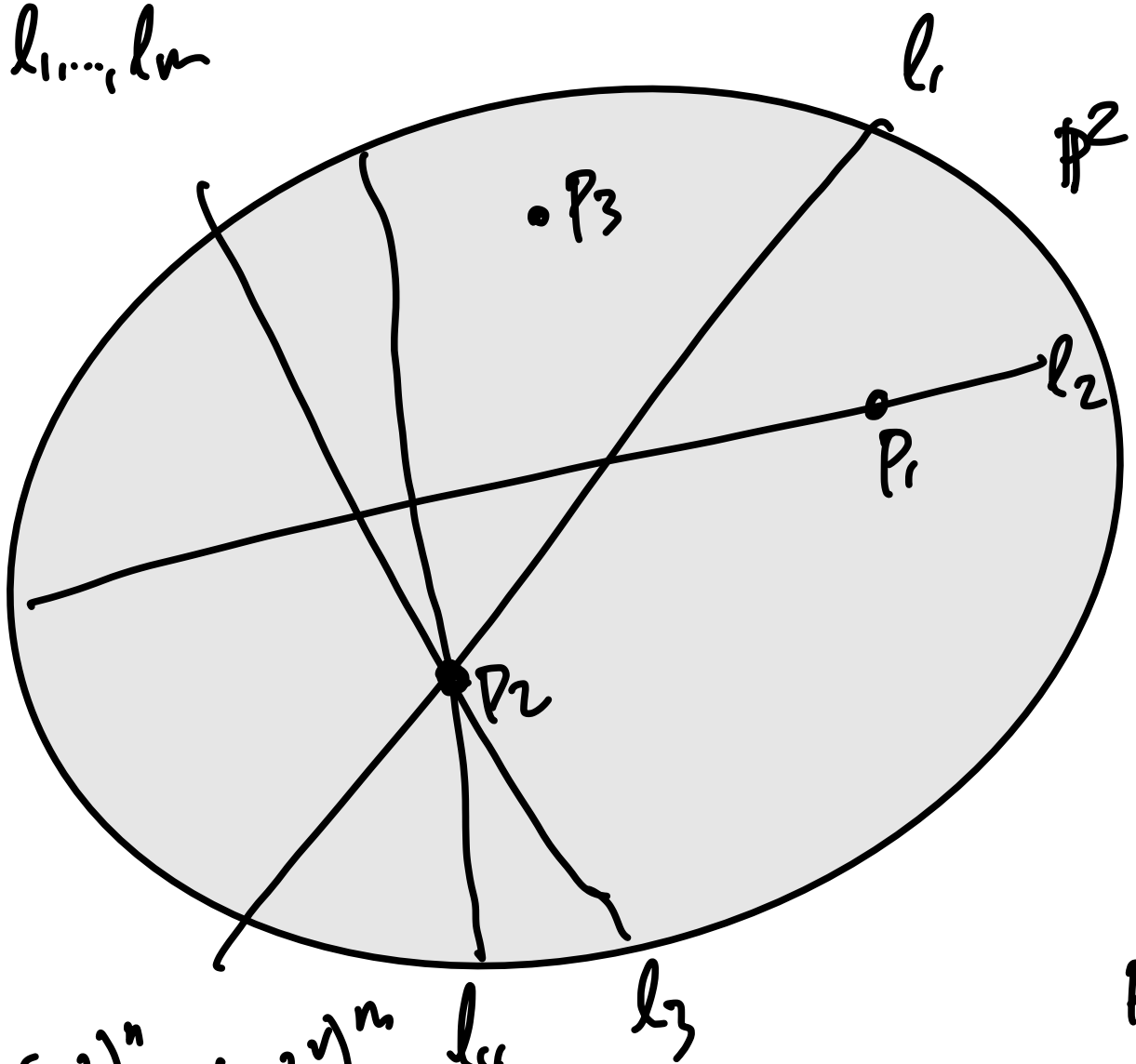
Thales.

m lines $l_1, \dots, l_m$

n points

$p_1, \dots, p_n$

distinct.



m n.

For each

$i \in \{1, \dots, m\}$

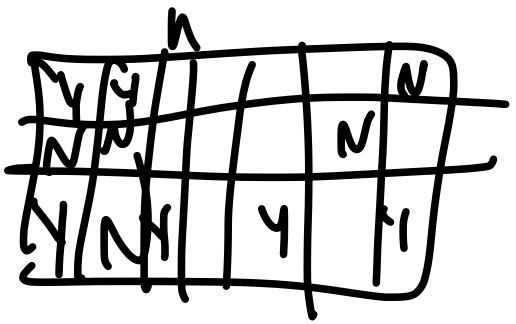
$j \in \{1, \dots, n\}$.

$p_j \in l_i$ or $p_j \notin l_i$

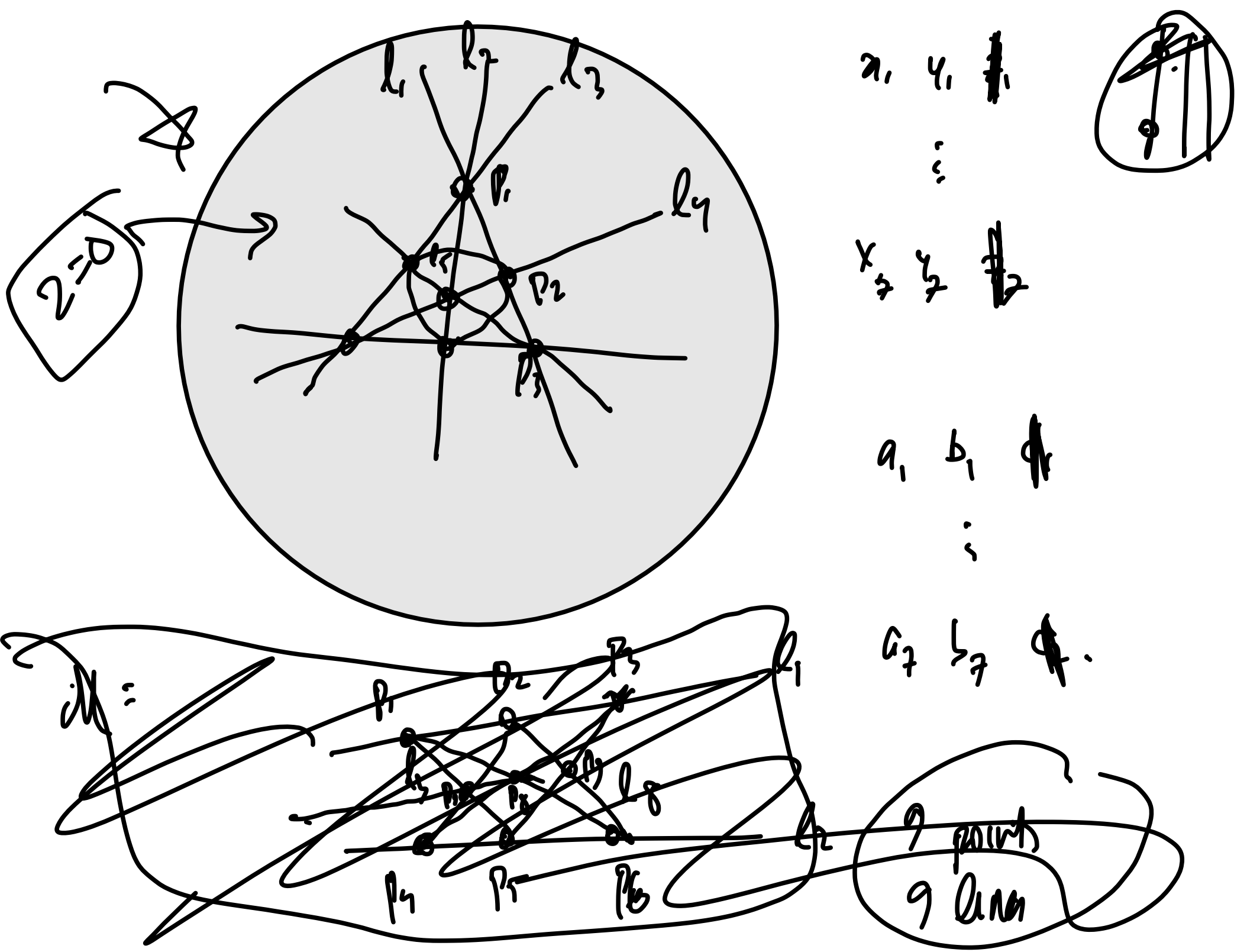
$C \subset (\mathbb{P}^2)^n \times (\mathbb{P}^{2\vee})^m$

x_j, y_j, z_j $a_i x + b_i y + c_i z = 0$

m



$a_i x_j + b_i x_j + c_i x_j \neq 0$


l_1
l_2
l_3
l_4
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
l_5
l_6
l_7
l_8
$z=0$
a_1 y_1 #
x_2 y_2 #
a_1 b_1 #
a_7 b_7 #.
9 points
9 lines

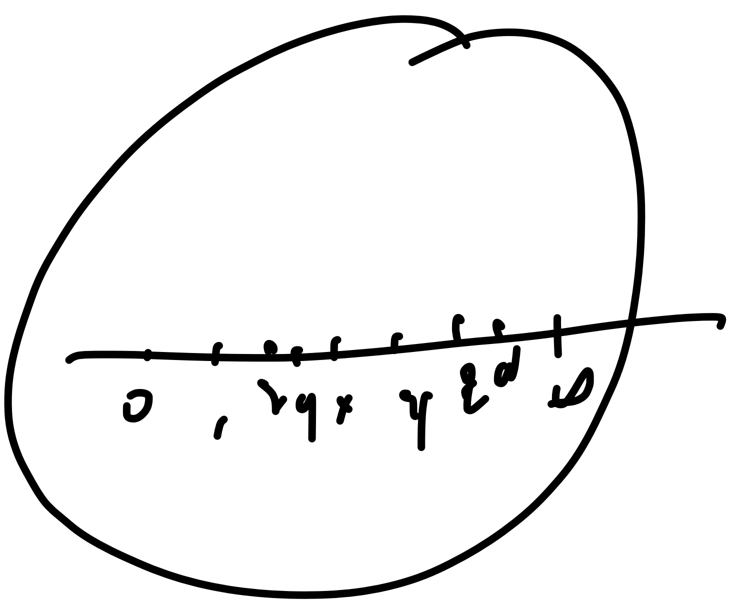
$$x^3 + 3x^2 w - 2yz + 17 = 4$$

$$3yw - 2z + 45(z+x)^{25} = 2$$

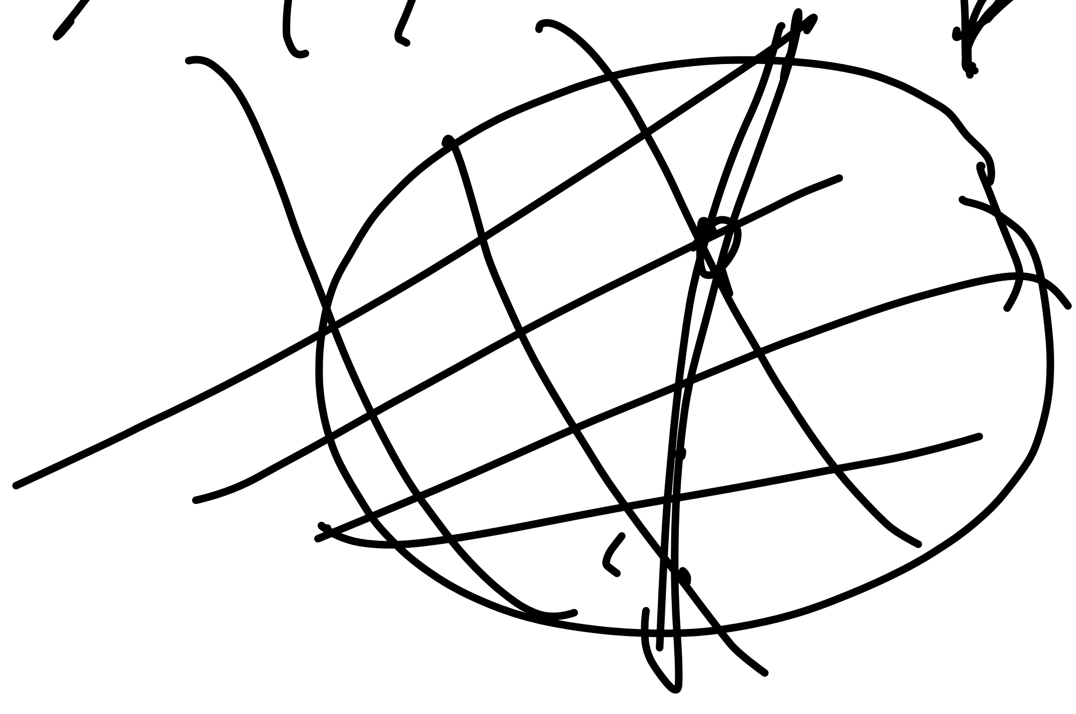
$$\vdots$$

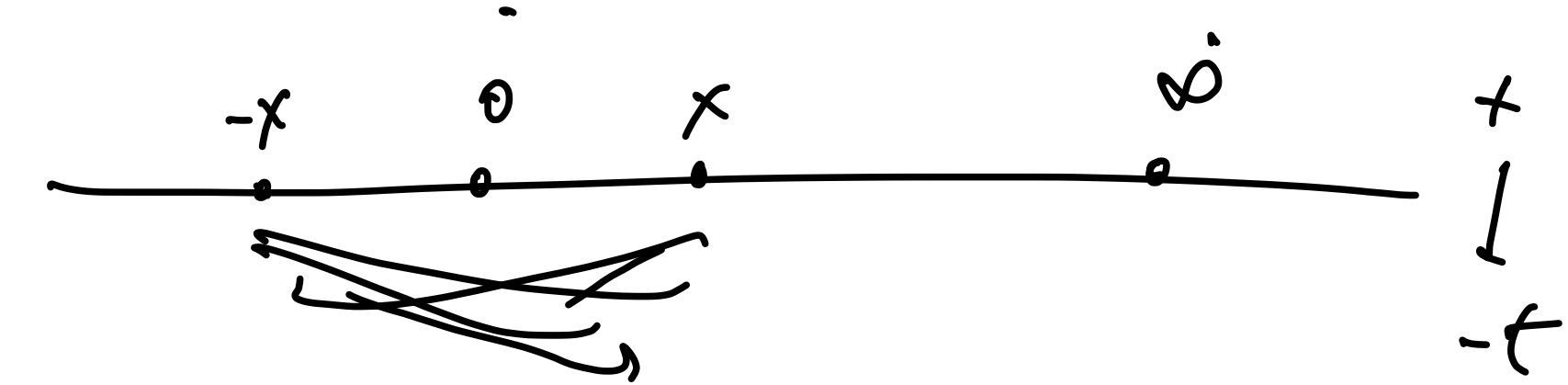
$(\ ) \in M$

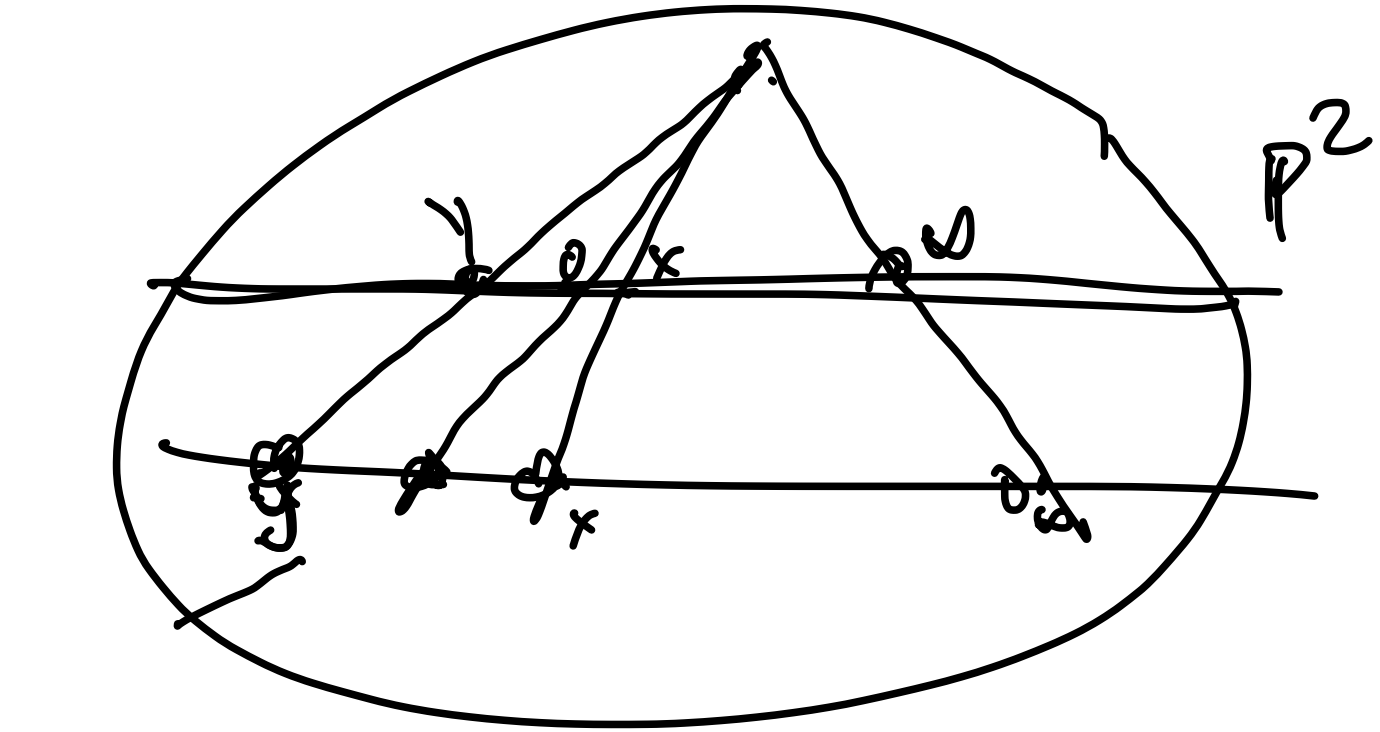
$$(x, y, z, w) = (\ldots)$$



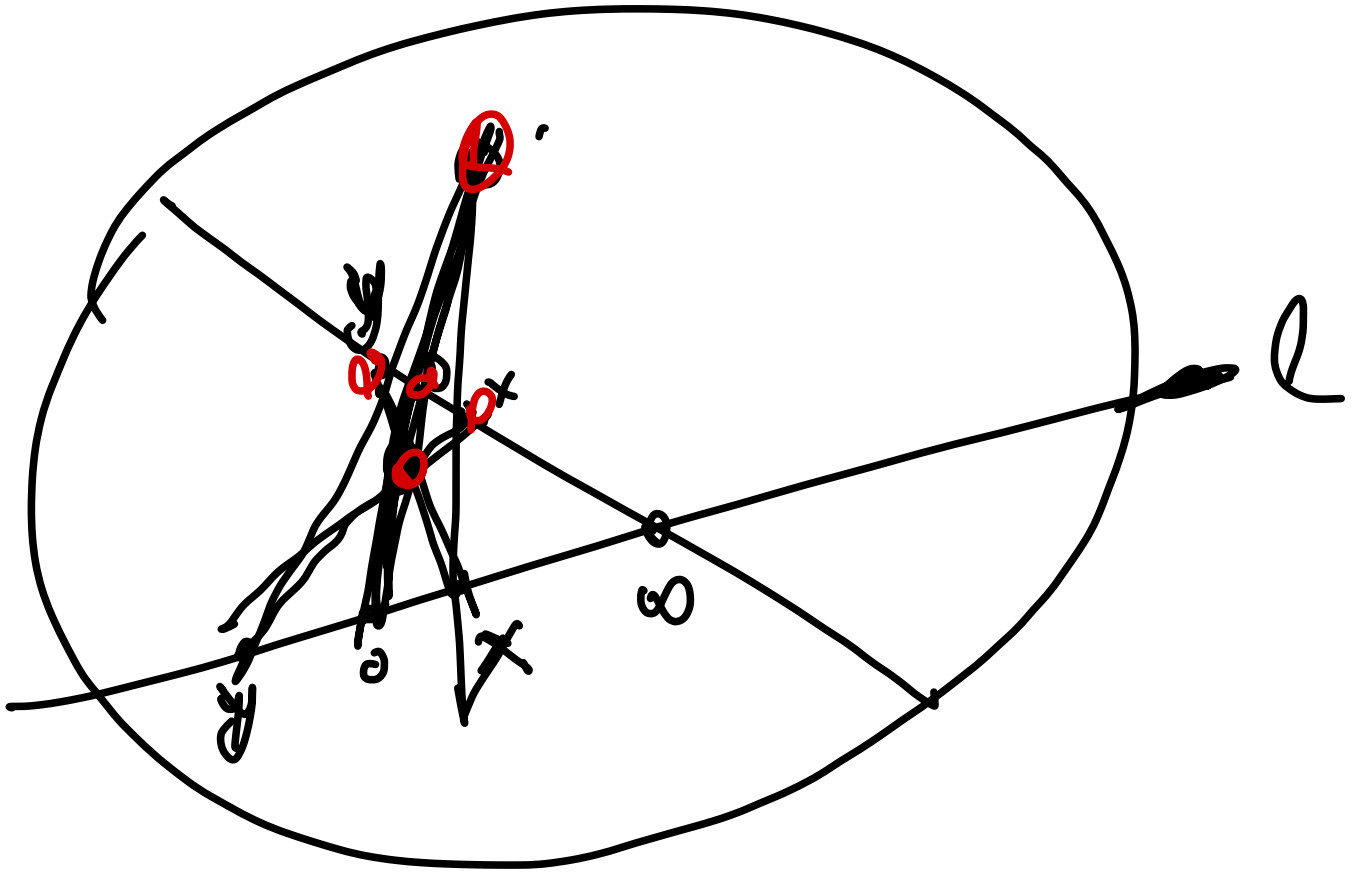
$\mathbb{R}^2$



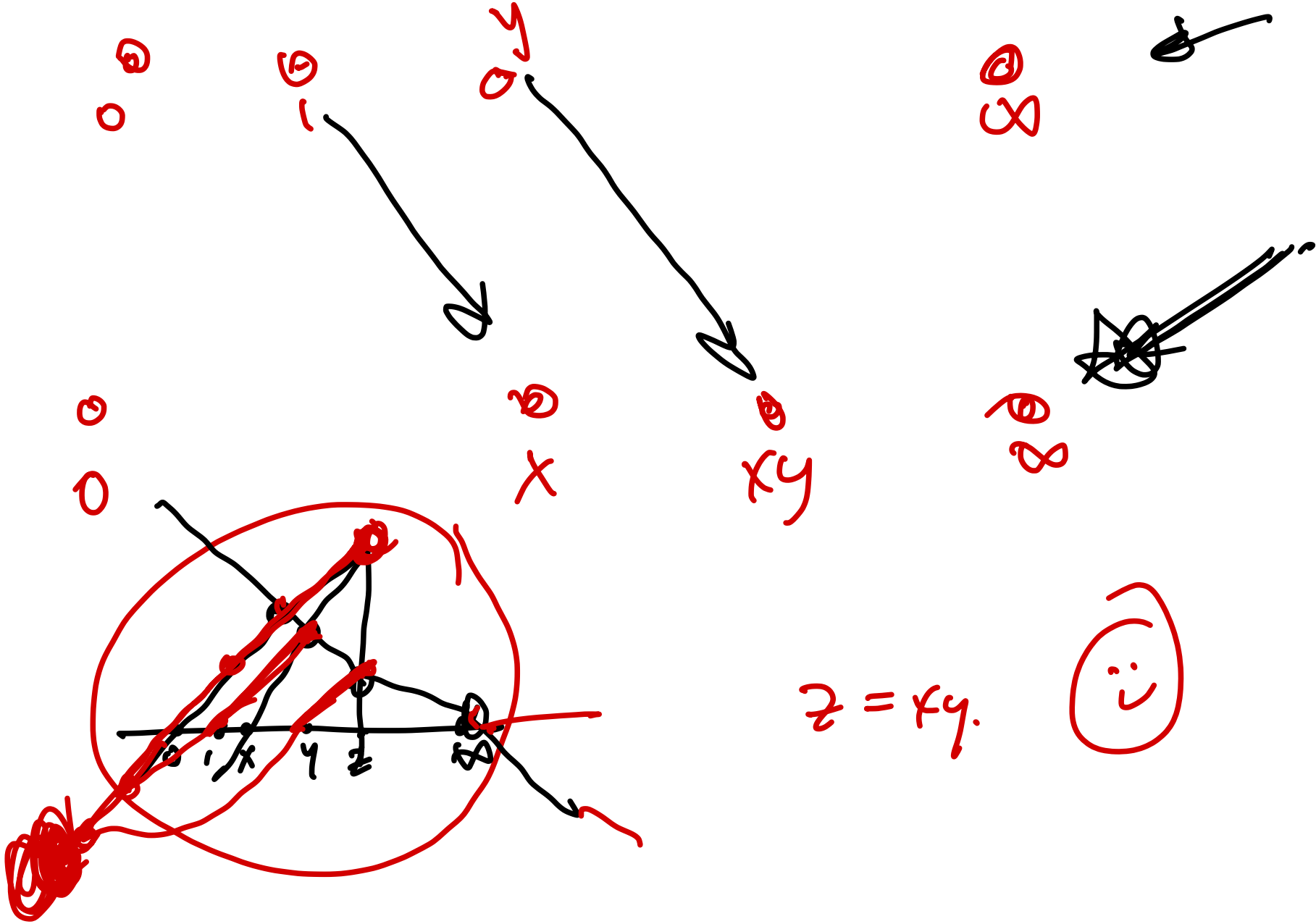

-x
0
x
+
-t


P^2
0
x

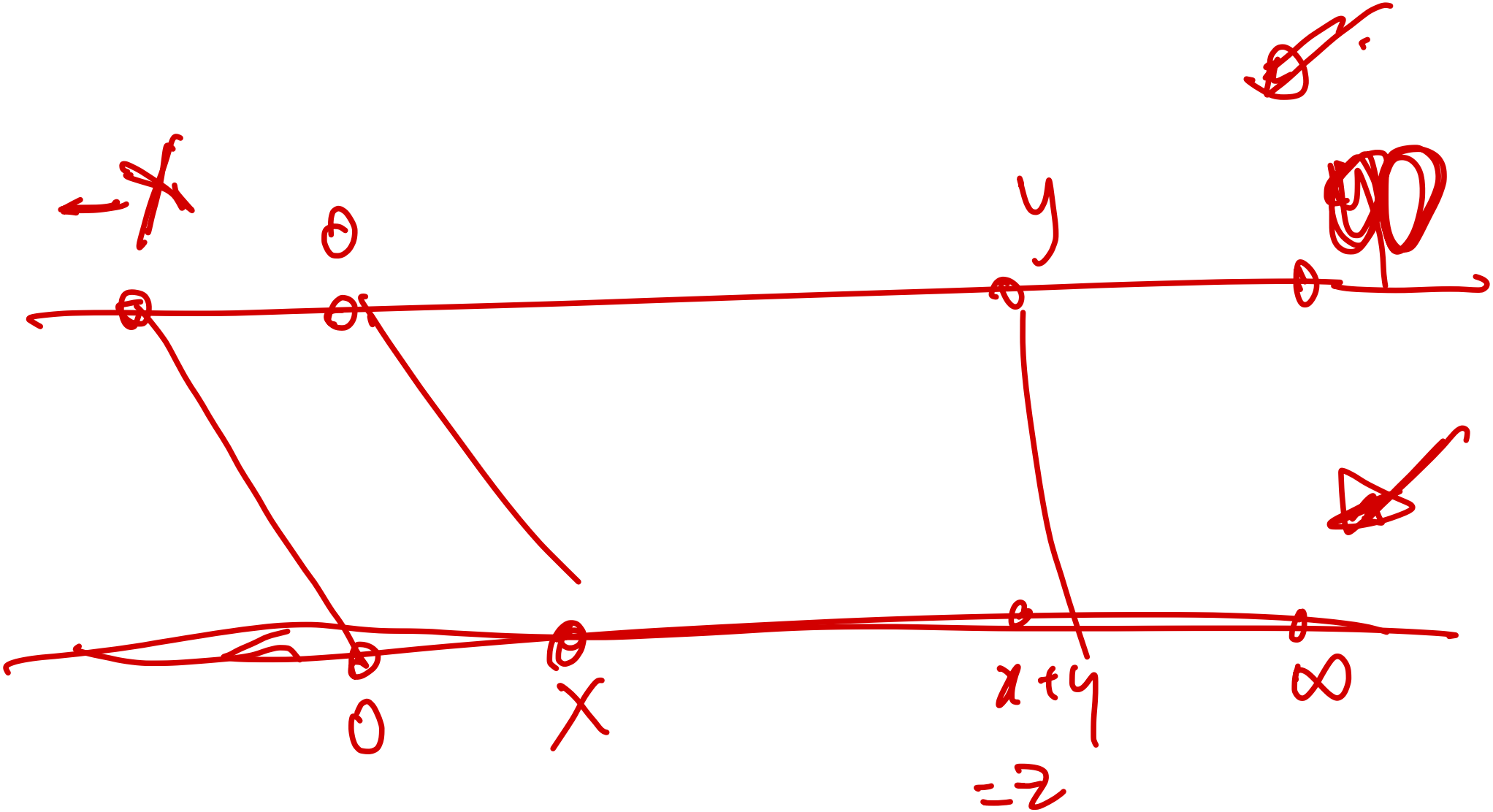
$X = -Y$

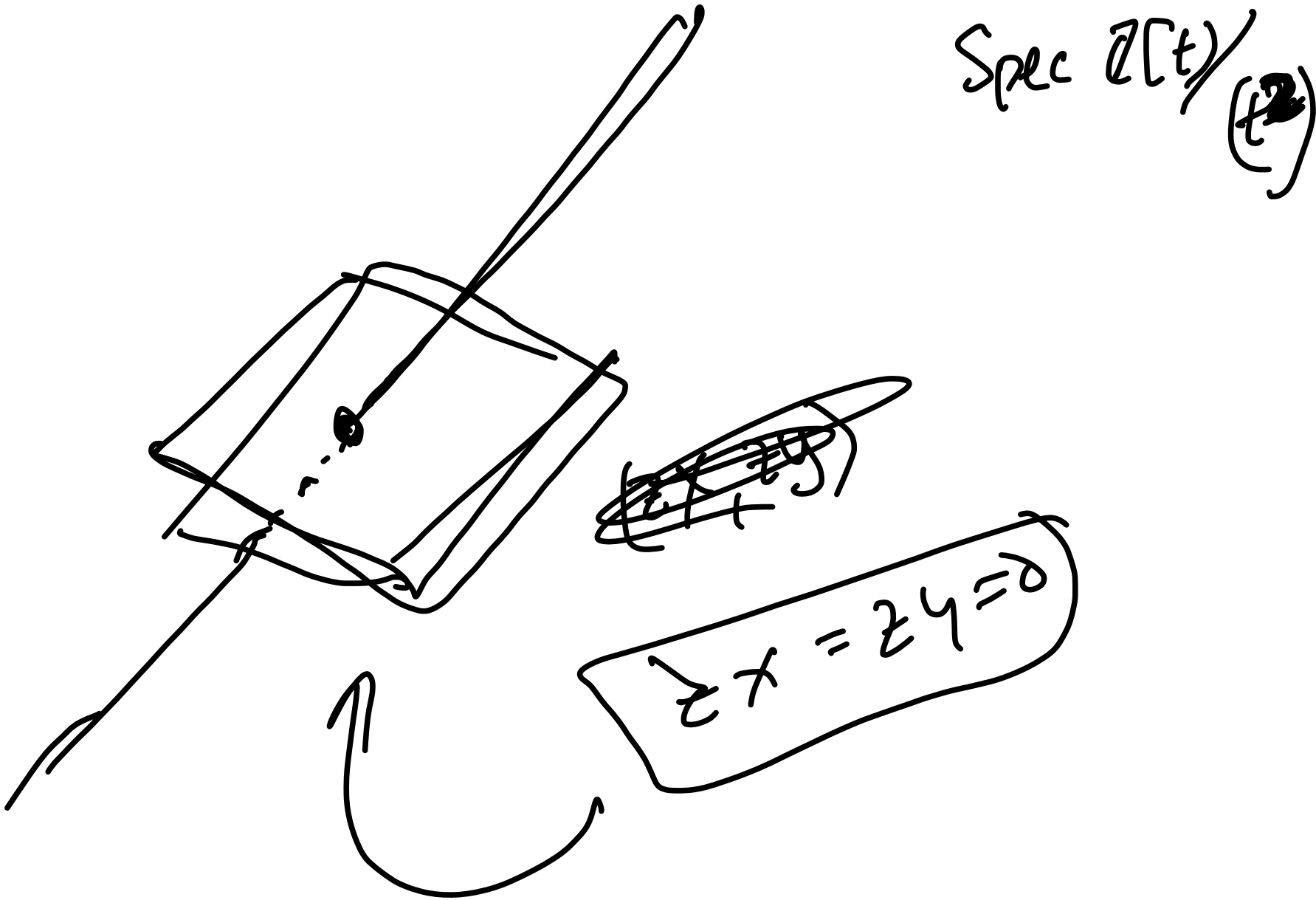


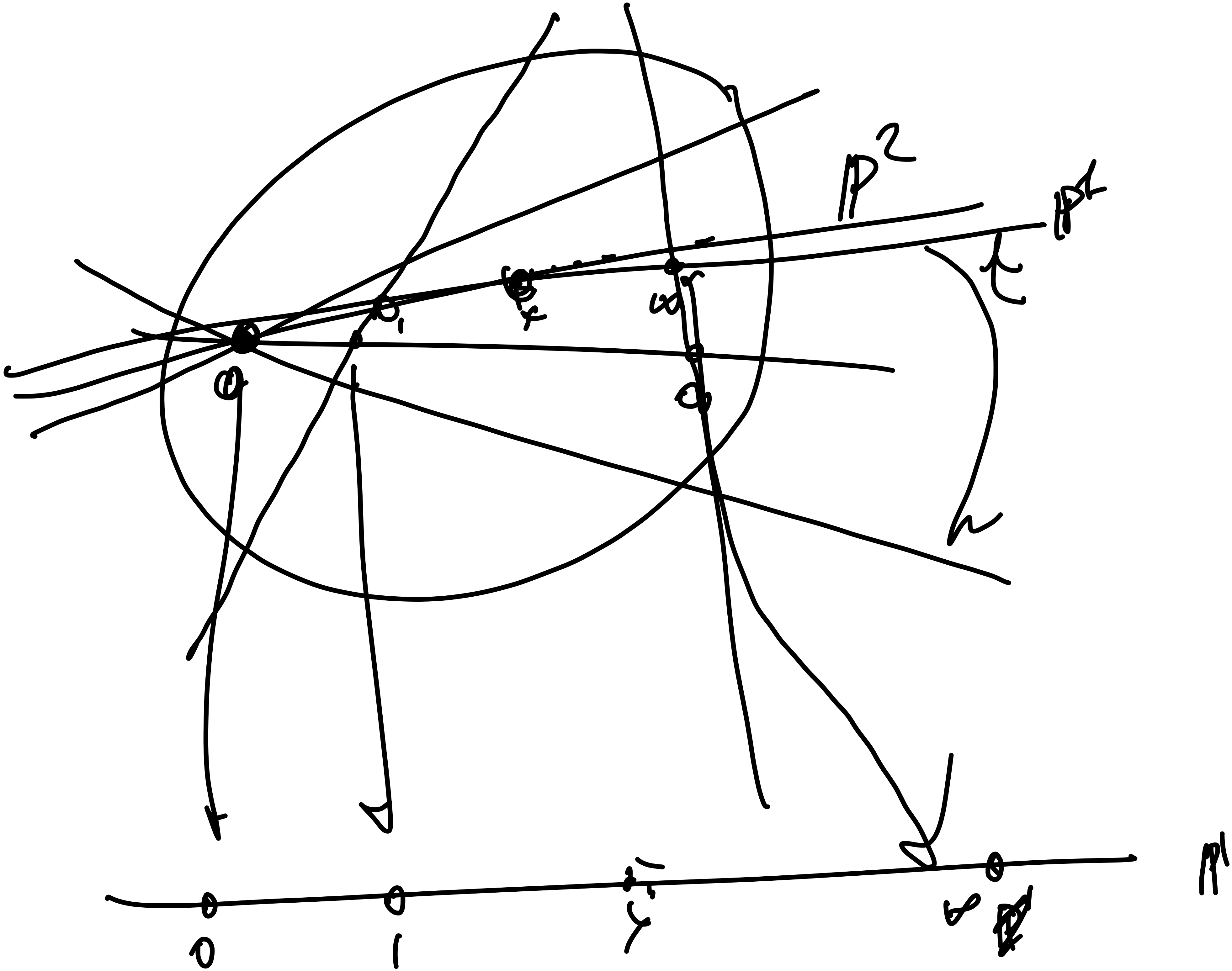
$$xy = z$$

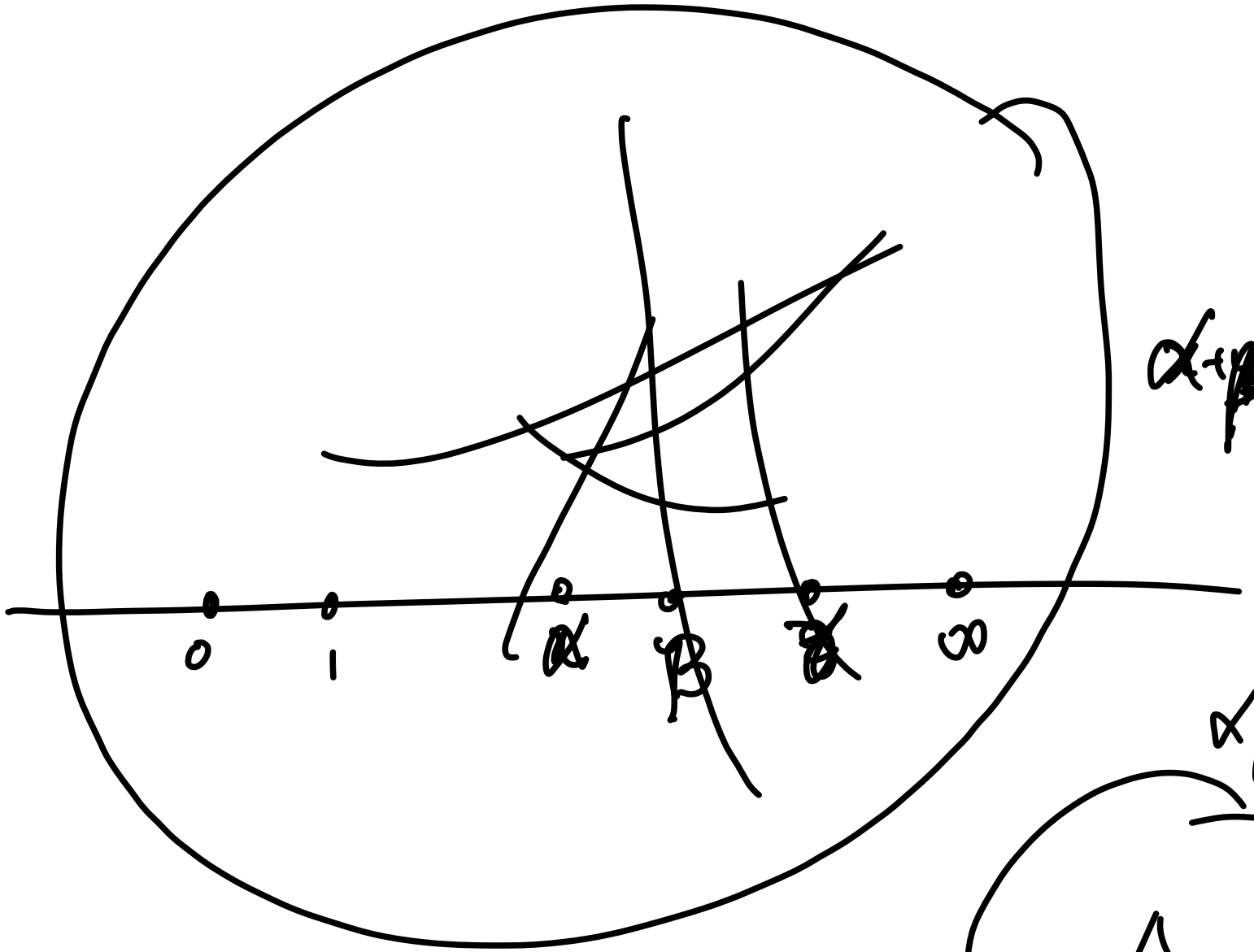


$z = xy.$


-x
0
y
0
x
x+y
∞
-z

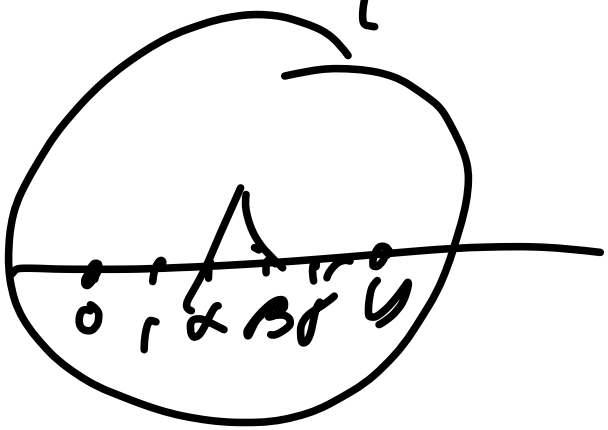

Spec ℤ[t]/(t²)
zx = zy = 0


$\mathbb{P}^2$
$\mathbb{P}^1$
0
1
∞



$\alpha + \beta =$

$\alpha\beta = \gamma$



$\alpha = -\beta$