18.034 MIDTERM 1

Explain your answers clearly; show all steps. Calculators may not be used. All problems have equal value. If you have any questions, please ask. GOOD LUCK!

1. (a) Find the modulus (absolute value) and argument of $1 + \sqrt{3}i$. Calculate $(1 + \sqrt{3}i)^6$.

(b) On the complex plane, sketch the location of the five solutions of $z^5 = 1$, i.e. the fifth roots of 1.

2. Consider the autonomous differential equation $y' = (y^2 - 1)e^y$. What are the equilibrium values of y? Which are stable and which are unstable? For which values of $y(0)$ will the solution stay bounded? What will the limiting value of y be in these cases? (Your answer will depend on $y(0)$.) Sketch the integral curves (in the xy-plane).

3. (a) Find the general solution of the differential equation $y' = (x + y)/x$. Hint: this equation is homogeneous, so try an appropriate substitution.

(b) For which constants k is

$$ke^x \cos y + (e^x \sin y - \sin y)y' = 0$$

an exact differential equation? In those cases, find the general solution ot the differential equation.

4. Consider the family of curves $y^2 - x^2 = c$, as c runs through the real numbers. Find another family of curves that is everywhere orthogonal to this one. (In other words, at almost every point in the plane, the tangent to the corresponding curve in the first family is perpendicular to the tangent to the corresponding curve in the second family.)

5. A not-uncommon calculus mistake is to believe that the product rule for derivatives says that $(fg)' = f'g'$. If $f(x) = x^2$, determine, with proof, whether there exists a nonzero function $g(x)$ defined on the set $(0, 2) = \{x|0 < x < 2\}$ such that the wrong product rule is true for x in $(0, 2)$.

6. Solve the differential equation $y'' + 4y = \cos(2.02t)$ with the initial conditions $y(0) = y'(0) = 0$. Sketch the solution. (Hint: you should see beats. How long are they, i.e. what is the period?) You may use the fact that $2.02^2 = 4.0804$, and $1/.0804 \cong 12.4$.

Date: March 10, 2000, 1:05-1:50 pm.