18.024 QUIZ III

(This quiz has two pages.) Write your name on the first page of your solutions. Time: 50 minutes. Justify all steps. If you have any questions, please ask. **GOOD LUCK!**

1. (16 points) Suppose $\vec{\alpha} : [0,1] \to \mathbb{R}^3$ given by $\vec{\alpha}(t) = (t, t^2, t^3)$, and $\vec{F}(x,y,z) = (0, y, x)$. Calculate $\int \vec{F} \cdot d\vec{\alpha}$.

2. (16 points) In (a) and (b), find if possible a potential function $\phi(x,y,z)$ defined on all of $\mathbb{R}^3$ for each of the following vector fields. If it is not possible, explain why not.

(a) $\vec{f}(x,y,z) = (2xy - y)\vec{i} + (y^2 + 2)\vec{j} + (3z)\vec{k}$
(b) $\vec{g}(x,y,z) = (y^2 + 2)\vec{i} + (2xy - y)\vec{j} + (3z)\vec{k}$
(c) Suppose $\vec{\alpha} : [0,1] \to \mathbb{R}^3$ given by $\vec{\alpha}(t) = ((e^t - 1)(t^2 - 1), \sin \pi t, 0)$. Calculate $\int \vec{g} \cdot d\vec{\alpha}$.

3. (16 points) If $\int\int_R f = \int_1^4 \int_{\sqrt{x}}^2 f(x,y)\, dy\, dx$, express $\int\int_R f$ as an iterated integral where the first (i.e. "inside") integration is with respect to x. Your answer should be of the form

$$\int_?^? \int_?^? ?\; dx\; dy.$$

4. (16 points) Express as an iterated integral the volume of the solid bounded by the surface $z = 2 - x^2 - y^2$ and the plane $z = 0$. (You don't need to evaluate the integral.)

5. (16 points) A piece of wire is bent into the circle $x^2 + y^2 = 4$ (of radius 2). The density of the wire is $10 + x + y$ (in units of mass per unit length). Find the mass of the wire, and the location $(\overline{x}, \overline{y})$ of its center of mass.

6. (20 points)

(a) Define what the statement "S has content zero" means.

Let $Q = [0,1] \times [0,1]$. Define $f(x,y)$ for (x,y) in Q by the equations

$$f(x,y) = \begin{cases} 2 & \text{if } y = x, \\ 1 & \text{if } x = 1/2 \text{ and } y \text{ is irrational}, \\ 0 & \text{otherwise.} \end{cases}$$

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(b) At what points does f fail to be continuous?

(c) Does $\int\int_Q f$ exist? Why or why not?

(d) Does $\int_0^1 f(x,y)dy$ exist for all x in $[0,1]$?

Bonus. (5 marks) Find the "hypervolume" of the n-dimensional version of a triangle (the n-simplex) in $\mathbb{R}^n$ bounded by $x_1 = 0$, $x_2 = 0$, $\ldots$, $x_n = 0$, $x_1 + x_2 + \cdots + x_n = r$.