18.024 QUIZ II

Write your name on the first page of your solutions. Time: 50 minutes. Justify all steps. If you have any questions, please ask. **GOOD LUCK!**

Here are some formulas (for reference) for velocity $\vec{v}$ and acceleration $\vec{a}$.

In terms of $\vec{T}$ and $\vec{N}$:

$$\begin{aligned} \vec{v} &= \left(\frac{ds}{dt}\right)\vec{T}, \\ \vec{a} &= \left(\frac{d^2s}{dt^2}\right)\vec{T} + \kappa\left(\frac{ds}{dt}\right)^2\vec{N}. \end{aligned}$$

In polar coordinates, in terms of $\vec{u}_r$ and $\vec{u}_\theta$:

$$\begin{aligned} \vec{v} &= \left(\frac{dr}{dt}\right)\vec{u}_r + \left(r\frac{d\theta}{dt}\right)\vec{u}_\theta, \\ \vec{a} &= \left(\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right)\vec{u}_r + \left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\vec{u}_\theta. \end{aligned}$$

1. (16 points)

(a) Set up the integral for the length of the curve given in Cartesian coordinates by the equation $y^3 = x^2$, for $-1 \le x \le 1$. Do not evaluate it!
(b) Set up the integral for the length of the curve given in polar coordinates by the equation $r = \sin\theta + \cos\theta$, for $0 \le \theta \le \pi/2$. Evaluate it. (Bonus 4 marks: the integral should have been very easy to evaluate. Why, geometrically, should the integrand have been so nice?)

2. (21 points) Given a scalar function f defined in a neighborhood of a point $\vec{a} \in \mathbb{R}^n$. Consider the following statements:

(i) f is continuous at $\vec{a}$.
(ii) f is differentiable at $\vec{a}$.
(iii) All directional derivatives $f'(\vec{a};\vec{y})$ exist at $\vec{a}$.
(iv) All partials $D_1 f, \ldots, D_n f$ exist in a neighborhood of $\vec{a}$ and are continuous at $\vec{a}$.

Answer "yes" or "no" (+3 points for each correct answer, -2 for each incorrect answer).

Date: Spring 2001.

- Does (ii) imply (i)? **Y/N**
- Does (ii) imply (iii)? **Y/N**
- Does (ii) imply (iv)? **Y/N**
- Does (iii) imply (ii)? **Y/N**
- Does (iii) imply (i)? **Y/N**
- Does (iv) imply (iii)? **Y/N**
- Does (iv) imply (ii)? **Y/N**

3. (16 points)

(a) Find a vector tangent to the curve of intersection of the surfaces $x^2 + 2y^2 + yz^2 = 1$ and $z = xe^{2y}$ at the point $P = (1, 0, 1)$.

(b) Find the directional derivative of

$$f(x, y, z) = x^2 + 2xy - yz$$

at the point $(1, 2, -1)$ in the direction pointing toward the origin.

4. (16 points) Suppose

$$\vec{f}(x, y) = (x^2 - x + y^2, e^x + y, \sin x + \cos y).$$

Calculate the derivative of $\vec{f}$ at $(0, 0)$ (hint: it's a matrix!), and estimate $\vec{f}(.001, .002)$ (i.e. give to three significant digits).

5. (15 points) If $g(s, t) = f(5s + t, 3st^2, s^2 - t)$, then find $\frac{\partial g}{\partial s}$ in terms of s, t and the partials $D_i f$ of f.

6. (16 points) The equation $x^2 + z + (y + z)^3 = 6$ defines z implicitly as a function of x and y, say $z = f(x, y)$. Find $\frac{\partial f}{\partial x}$ in terms of x, y, and z.