18.024 PRACTICE QUIZ I

1. (20 points) Let L_1 be the line through the point $P = (a, 0, 0)$ on the x-axis with direction vector $(-3, 1, -1)$. Let L_2 be the line $X = (1, 2, 0) + t(1, -1, 2)$. If L_1 and L_2 intersect, find the point P.

2. (24 points) Let A be a k by n matrix; let r be the rank of A. Answer the following questions in terms of n, k, and r. (Give answers only.)

(a) What can you say about the dimension of the row space of A?
(b) What can you say about the dimension of the solution space of the equation $AX = 0$?
(c) What can you say if the system $AX = C$ fails to have a solution for some C?
(d) What can you say if you know A has an inverse?

3. (20 points) Find conditions on a, b, c that are both necessary and sufficient for the following system to have a solution.

$$\begin{array}{ccccccc} & & y & - & 2z & = & a \\ x & - & y & + & z & = & b \\ x & + & y & - & 3z & = & c \end{array}$$

4. (20 points) Find the inverse of the matrix

$$A = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

5. (16 points) Let A be a 5 by 5 matrix. Show that if A^3 has rank less than 5, then A has rank less than 5.

Another tricky question: Suppose A, B, and C are three vectors in V_5. Can $3A+2B+4C$, $A+4B+2C$, $9A+4B+3C$, and $A+2B+5C$ be linearly independent?

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