MATH 51H HOMEWORK 4 (due Friday, October 26)

These are from volume II. You should do all the problems, but you should hand in only the two starred text problems from the following list, together with the 3 extra problems.

P. 57: Do some of the exercises if you need a review of matrix multiplication. (Do not hand in.)

P. 246: 2(a-h) (do not hand in), 5*
P. 251: 1(a, b, f, h), 6*

(Note: answers to many of the text problems are in the back of the book.)

Extra problems:

1. For \( \mathbf{p} \in \mathbb{R}^3 \), let \( F(\mathbf{p}) \) be the image of \( \mathbf{p} \) under reflection in the plane \( x = y \). (In other words, the plane \( x = y \) is the perpendicular bisector of the segment from \( \mathbf{p} \) to \( F(\mathbf{p}) \).) Find the \( 3 \times 3 \) matrix \( A \) such that

\[
F(\mathbf{p}) = A\mathbf{p}
\]

for every \( \mathbf{p} \in \mathbb{R}^3 \).

2. Let \( \mathbf{a} = (a_1, a_2, a_3) \) be a vector in \( \mathbb{R}^3 \). Consider the map \( G : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) defined by

\[
G(\mathbf{v}) = \mathbf{a} \times \mathbf{v}.
\]

Find the \( 3 \times 3 \) matrix \( P \) such that \( G(\mathbf{v}) = P\mathbf{v} \) for every \( \mathbf{v} \in \mathbb{R}^3 \).

3. Given \( \mathbf{v} \in \mathbb{R}^3 \), let \( H(\mathbf{v}) \) be the point obtained from \( \mathbf{v} \) by rotating it by 120° about the line \( x = y = z \). (There are two directions you can rotate: choose either one you like.) Find the \( 3 \times 3 \) matrix \( R \) such that \( H(\mathbf{v}) = R\mathbf{v} \) for every \( \mathbf{v} \in \mathbb{R}^3 \).