HW 2 - Solutions of the Rest of the Problems

4.1.3 (Page 113)

The fixed points are at \( \sin 2\theta = 0 \), i.e. at \( \theta = 0, \pi/2, \pi, 3/2\pi \). The vector field looks as shown below:

![Diagram for 4.1.3]

4.1.4 (Page 113)

The fixed points are at \( \sin^3 \theta = 0 \), i.e. at \( \theta = 0, \pi \). The vector field looks as shown below:

![Diagram for 4.1.4]

4.3.5. (Page 115)

The phase portraits are vertical translates (by \( \mu \)) of the graph of \( \cos \theta + \cos 2\theta \) which is presented below. All bifurcations are of saddle node type. If \( \mu > 9/8 \) or \( \mu < -2 \) there are no critical points. If \(-2 < \mu < 0\) we have one stable and one unstable point, if \( 0 < \mu < 9/8 \) then there are two stable and two unstable points. The bifurcation diagram is also shown below.
We get the following phase portraits and types of vector fields for several different values of \( \mu \).
Graph of $\theta = \frac{\sin \theta}{\mu + \sin \theta}$

$\mu > 1$ (case $\mu = 2$):

$\mu = 1$:

$\mu = 1/2$:

Vector field
For $\mu \notin [-1, 1]$ we have one stable and one unstable critical point. For $\mu = 1$ or $\mu = -1$ we get a saddle-node bifurcation since two blow-up points appear, one stable and one unstable. For $\mu \in (-1, 1), \mu \neq 0$ we get two critical points and two blow-up points. Notice at $\mu = 0$ there are no critical points and the behaviour for the possibilities $\mu < 0$ is symmetric to the already described cases. As $\mu$ passes through 0 the blow-ups cross the critical points and each blow-up exchanges its stability with the critical point it passes. This is analogous to two transcritical bifurcations. This allows us to draw the following bifurcation diagram: