1. Suppose $F : \mathbb{Q} \to \mathbb{Q}$ is an homomorphism. Then 

\[ (*) \quad F(p \cdot x) = p \cdot F(x) \]

for any integer $p$ and any $x \in \mathbb{Q}$ (see note). In particular, if $p \neq 0$, then (*) holds for $x = 1/p$:

\[ F(1) = p \cdot F(1/p) \]

or 

\[ (**) \quad F(1/p) = (1/p) \cdot F(1). \]

Now any rational number $z$ can be written as $n/p$ for integers $n$ and $p$ with $p \neq 0$. Then by (*) and (**):

\[ F(z) = F(n \cdot (1/p)) = n \cdot F(1/p) = (n/p) \cdot F(1) = F(1) \cdot z. \]

so $F(z) = az$ where $a = F(1)$. Of course if $F$ is an automorphism, then this map must be a bijection, which implies that $a \neq 0$.

Note: The statement (*) may look more familiar when we write it for a multiplicative group: $F(x^p) = F(x)^p$. 