3.5.12. The idea is: write any $\sigma \in S_{n-2}$ as a product of disjoint cycles. If it is even, add the cycles $(n-1)(n)$ at the end to get an element of $A_n$. If it is odd, add the cycle $(n-1 \ n)$ at the end to get an element of $A_n$. Of course we have to check that this defines an injective homomorphism.

More formally, for $\sigma \in S_{n-2}$, define $\sigma' \in S_n$ as follows:

$$\sigma'(k) = \begin{cases} 
\sigma(k) & \text{if } k \leq n-2, \text{ and} \\
k & \text{if } k = n-2 \text{ or } k = n.
\end{cases}$$

Of course $\sigma \mapsto \sigma'$ is an injective homomorphism from $S_{n-2}$ to $S_n$. But of course $\sigma'$ need not be in $A_n$.

Define another homomorphism $f : S_{n-2} \to S_n$ as follows:

$$f(\tau) = \begin{cases} 
(n-1 \ n) & \text{if } \tau \text{ is odd, and} \\
1 & \text{if } \tau \text{ is even.}
\end{cases}$$

Now let $g(\sigma) = \sigma'f(\sigma)$. Clearly $g$ maps $S_{n-2}$ to $A_n$, so we just have to check that it is a homomorphism and that it is injective.

Note that if $\sigma \in S_{n-2}$, then $\sigma'$ only affects the numbers $1, \ldots, n-2$ whereas $f(\sigma)$ only affects $n-1$ and $n$. Thus if $\sigma$ and $\tau$ are in $S_{n-2}$, $f(\tau)$ commutes with $\sigma'$. Thus:

$$g(\sigma)g(\tau) = \sigma'f(\sigma)\tau'f(\tau)$$
$$= \sigma'\tau'f(\sigma)f(\tau)$$
$$= (\sigma\tau)'f(\sigma\tau) \quad \text{(since }' \text{ and } f \text{ are homomorphisms)}$$
$$= g(\sigma\tau)$$

so $g$ is a homomorphism. Also, $g(\sigma)(i) = \sigma(i)$ for $i \leq n-2$. So $\sigma$ is determined by $g(\sigma)$. That is, $g$ is injective. $\square$