1. For any positive integer \( n \), let \( \langle n \rangle \) denote the closest integer to \( \sqrt{n} \). Evaluate

\[
\sum_{n=1}^{\infty} \frac{2^{\langle n \rangle} + 2^{-\langle n \rangle}}{2^n}.
\]

(Nathan Pflueger, 2001B3)

2. Let \( a, b, c, d \) be integers such that \( a > b > c > d > 0 \). Suppose that

\[
ac + bd = (b + d + a - c)(b + d - a + c).
\]

Prove that \( ab + cd \) is not prime. (Kiat Chuan Tan, IMO2001 # 6)

3. Show that the set \( 1, 2, \ldots, 2^n \) can be partitioned into two subsets which contain no arithmetic progressions of length \( 2n \). (Dragos Oprea, from an old Romanian Olympiad)

4. Let

\[
\begin{align*}
&\begin{array}{cccc}
a_{1,1} & a_{1,2} & a_{1,3} & \ldots \\
a_{2,1} & a_{2,2} & a_{2,3} & \ldots \\
a_{3,1} & a_{3,2} & a_{3,3} & \ldots \\
\vdots & \vdots & \vdots & \ddots
\end{array}
\end{align*}
\]

be a doubly infinite array of positive integers, and suppose each positive integer appears exactly eight times in the array. Prove that \( a_{m,n} > mn \) for some pair of positive integers \( (m, n) \). (Siddhartha, 1985B3)

5. Prove that there is a constant \( C \) such that, if \( p(x) \) is a polynomial of degree 1999, then

\[
|p(0)| \leq C \int_{-1}^{1} |p(x)| \, dx.
\]

(Bob Hough, 1999A5)

6. Find all pairs of real numbers \( (x, y) \) satisfying the system of equations

\[
\begin{align*}
\frac{1}{x} + \frac{1}{2y} &= (x^2 + 3y^2)(3x^2 + y^2) \\
\frac{1}{x} - \frac{1}{2y} &= 2(y^4 - x^4).
\end{align*}
\]

(Nathan Pflueger, 2001B2)

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