The William Lowell Putnam Mathematical Competition

takes place Saturday, December 7, 2002.
Last year, Stanford, placed fifth.

Sign-up and Introductory Meeting

We will also discuss times and dates of problem-solving preparatory sessions. If you can’t
make it and are even potentially interested, please e-mail vakil@math.stanford.edu.
For more information: http://math.stanford.edu/~vakil/stanfordputnam

Sample problems:
1. Let $S$ be a set of real numbers which is closed under multiplication (that is, if $a$ and $b$ are
in $S$, then so is $ab$). Let $T$ and $U$ be disjoint subsets of $S$ whose union is $S$. Given that the
product of any three (not necessarily distinct) elements of $T$ is in $T$ and that the product
of any three elements of $U$ is in $U$, show that at least one of the two subsets $T, U$ is closed
under multiplication.
2. Inscribe a rectangle of base $b$ and height $h$ and an isosceles triangle of base $b$ in a circle of
radius one as shown. For what value of $h$ do the rectangle and triangle have the same area?

3. Evaluate
\[ \int_2^4 \frac{\sqrt{\ln(9 - x)}}{\sqrt{\ln(9 - x) + \ln(x + 3)}} \, dx. \]

4. Find all real-valued continuously differentiable functions $f$ on the real line such that for
all $x$
\[ (f(x))^2 = \int_0^x ((f(t))^2 + (f'(t))^2) \, dt + 1990. \]

5. Prove that, for any integers $a, b, c$, there exists a positive integer $n$ such that $\sqrt{n^3 + an^2 + bn + c}$
is not an integer.