MODERN ALGEBRA (MATH 210) PROBLEM SET 5

The proof of the Zassenhaus Lemma (p. 279), and hence the Jordan-Hölder Theorem, relies on the certain facts unproved in the text. Rotman defines a map of sets

\[ \phi : A(A^* \cap B^*) \to \frac{A^* \cap B^*}{(A \cap B^*)(A^* \cap B)} \]

by \( ax \mapsto xD \), where \( a \in A \) and \( x \in A^* \cap B^* \). Show that (i) \( \phi \) is a group homomorphism, (ii) \( \phi \) is surjective, and (iii) \( \text{Ker} \phi = A(A^* \cap B) \).

Do nine of the following ten problems from the text: 5.18 , 5.33 , 5.34 , 5.37 , 5.39 , 5.40 , 5.41 , 5.42 , 5.45 , 5.49 .

The set is due Tuesday, November 19 at 3:30 pm in Pierre Albin’s mailbox.