1.) Presenting your argument in steps, using only the axioms of an ordered field (listed as Axioms 1-12 in the text, on pages 10 - 12) and stating which property is used at each step, prove the following:

i) \((-1)a = -a\).

ii) \(a > 0 \Rightarrow \frac{1}{a} > 0\).

iii) The complex numbers, \(\mathbb{C}\), do not form an ordered field; that is, there is no subset \(P \subset \mathbb{C}\) such that Axiom 12 holds.

2.) Prove that every positive real number \(a > 0\) has a positive square root. (Hint: Let \(S = \{x \in \mathbb{R} : x > 0 \text{ and } x^2 < a\}\), and begin by showing that \(S\) is non-empty and bounded above.)

3.) Exercise 5.1 on page 16 of the text.

4.) Exercise 5.7 on page 17 of the text.

5.) Exercise 7.9 on page 24-25 of the text. The asterisk means that there is a hint at the end of the book.

6.) Exercise 16.3 on page 53 of the text.

7.) Exercise 16.8 on page 54 of the text.

8.) Exercise 19.3 on page 61 of the text.

9.) Consider the set of rational numbers. Call two Cauchy sequences of rationals \(\{a_n\}\) and \(\{b_n\}\) equivalent, denoted \(\{a_n\} \sim \{b_n\}\), if \(\lim_{n \to \infty} |a_n - b_n| = 0\).

   a) Show that \(\sim\) is an equivalence relation (symmetric, reflexive, and transitive).

   b) Define \(\mathcal{R}\) to be the set of equivalence classes of Cauchy sequences of rationals. If \(x, y \in \mathcal{R}\), define \(x + y\) and \(xy\), showing that your definition is independent of the choices within an equivalence class.

10.) Let \(\mathcal{R}\) be defined as in the previous problem.

   a) Show that \(\mathcal{R}\) satisfies field Axioms 5 and 10: the existence of additive and multiplicative inverses. (It satisfies the other field axioms but you don’t have to prove that.)

   b) Describe a set \(P \subset \mathcal{R}\) of positive elements and show that it satisfies the order axiom, Axiom 12.