You have 50 minutes to complete this exam.

No notes, books, calculators, or other references are allowed.

You must show all work to receive credit. Answers for which no work is shown will receive no credit (unless specifically stated otherwise).

There are problems on the front and back of each page. Make sure that you answer all of the questions. There is extra paper available if you need it.

Good luck! Have fun! Eat candy as necessary.
1. (7 points) For what value(s) of $h$ are the vectors

$$
\begin{bmatrix}
1 \\
2 \\
1
\end{bmatrix}, \begin{bmatrix}
2 \\
5 \\
3
\end{bmatrix}, \begin{bmatrix}
-1 \\
-4 \\
h
\end{bmatrix}
$$

linearly dependent? Make sure that you justify your response and show all work. An answer (even a correct one) with no work shown will earn no credit.
2. (7 points) Consider the matrix \( A = \begin{bmatrix} 1 & 5 & 4 & 3 & 2 \\ 1 & 6 & 6 & 6 & 6 \\ 1 & 7 & 8 & 10 & 12 \\ 1 & 6 & 6 & 7 & 8 \end{bmatrix} \). You may use the fact that \( \text{rref}(A) = \begin{bmatrix} 1 & 0 & -6 & 0 & 6 \\ 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \). Write each non-pivot column of \( A \) as a linear combination of the pivot columns of \( A \).
3. (9 points total, 3 points each)

(a) Suppose that $A$ is a $3 \times 3$ matrix such that $\det(A) = 7$. Find, with explanation, $\det(A^2 A^T A^{-1})$.

(b) Suppose that $A$ is a $5 \times 8$ matrix. What is the minimum number of free variables in the solution space of $Ax = 0$?
(c) Suppose that $A$ is a $5 \times 9$ matrix such that

\[
\begin{align*}
\begin{bmatrix}
7 & -3 & 1 \\
0 & 1 & -2 \\
4 & 2 & 0 \\
1 & -6 & 3 \\
2 & 8 & 1 \\
\end{bmatrix}
\end{align*}
\]

is a basis for the column space of $A$. Find $m, n, p, q$ so that the following statement is true: The column space of $A$ is an $m$-dimensional subspace of $\mathbb{R}^n$ and the nullspace of $A$ is a $p$-dimensional subspace of $\mathbb{R}^q$. No explanation is necessary.

\[
m = \\
n = \\
p = \\
q = \\
\]
4. (12 points total, 4 points each)
   (a) Find the general solution of the differential equation
   \[ y'' + 8y' - 9y = 0. \]
   (b) Find the general solution of the differential equation
   \[ y'' + 8y' - 9y = 9x. \]
(c) Find the general solution of the differential equation

\[ y'' + 8y' - 9y = 3e^x. \]
5. (5 points) Find the general solution of the differential equation
\[ y^{(4)} - 16y = 0. \]

. Hint for factoring the characteristic polynomial: \( a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) \).
6. (5 points) Find a second-order, linear, homogeneous differential equation with constant coefficients for which the function

\[ y = 5 - 8e^{-3x} \]

is a solution, or explain why it’s not possible to find such a differential equation.
7. (5 points) Determine whether each of the following statements is True or False. These questions will be scored as follows: +1 points for a correct answer, 0 points for no response, and $-1/2$ points for an incorrect answer. No explanation is necessary.

(a) The vectors \[
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
\end{bmatrix}, \quad \begin{bmatrix}
n & 4 & \pi \\
-3 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}, \quad \begin{bmatrix}
0 \\
0 \\
1 \\
0 \\
\end{bmatrix}, \quad \begin{bmatrix}
1 \\
1 \\
1 \\
2 \\
\end{bmatrix}, \quad \begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
\end{bmatrix}
\] span $\mathbb{R}^4$.

(b) The vectors \[
\begin{bmatrix}
1 \\
2 \\
\end{bmatrix}, \quad \begin{bmatrix}
3 \\
4 \\
\end{bmatrix}
\] form a basis for $\mathbb{R}^2$.

(c) The general solution of the differential equation $y'' - 6y' + 13y = 10e^{4x}$ is $y = 2c_1e^{4x}$, where $c_1$ is any constant.

(d) Every set of 3 vectors in $\mathbb{R}^3$ spans $\mathbb{R}^3$.

(e) If $y_1$ and $y_2$ are two solutions of the differential equation $y'' - 5y' + 6y = 10$, then $y_1 + y_2$ is also a solution.