Math 244 Exam 1  
Tuesday, April 27, 2010

Name:

- You have 50 minutes to complete this exam.
- No notes, books, calculators, or other references are allowed.
- You must show all work to receive credit. Answers for which no work is shown will receive no credit (unless specifically stated otherwise).
- There are problems on the front and back of each page. Make sure that you answer all of the questions. There is extra paper available if you need it.
- Good luck! Have fun! Eat candy as necessary.

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1. (10 points) Find the solution of the initial-value problem

\[ xy' - y = x^2 + \frac{4}{x}, \quad y(2) = 9. \]
2. (Note that this problem has 2 parts. Part (b) is on the next page.)

(a) (5 points) Find all solutions of the linear system

\[
\begin{align*}
  x_1 - 2x_2 + x_3 - x_4 &= 0 \\
  2x_1 - 3x_2 + 4x_3 - 3x_4 &= 0 \\
  3x_1 - 5x_2 + 5x_3 - 4x_4 &= 0 
\end{align*}
\]

Write your answer in vector (or parametric) form.
(b) (3 points) Find a non-zero vector $\mathbf{x}$ such that

$$
\begin{bmatrix}
1 & -2 & 1 & -1 \\
2 & -3 & 4 & -3 \\
3 & -5 & 5 & -4
\end{bmatrix}
\mathbf{x} = \mathbf{0}.
$$
3. (6 points total, 3 points each) Use the following matrices for this problem:

\[
A = \begin{bmatrix} 1 & -4 \\ 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ 1 & 7 \\ 0 & -2 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}.
\]

(a) Which of \(AB\) or \(BA\) is defined? Compute the product that is defined.

(b) Compute \(B^T x\), or state that the product is not defined.
4. (4 points) Find the determinant of the matrix

\[ A = \begin{bmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{bmatrix} \]
5. (a) (2 points) Suppose that $A$ is a $4 \times 2$ matrix, $B$ is a $2 \times 2$ matrix, and $X$ is a matrix such that

$$A = XB.$$ 

What is the size of $X$?

(b) (6 points) Find a matrix $X$ such that

$$\begin{bmatrix}
1 & 0 \\
2 & -1 \\
0 & 3 \\
4 & 1
\end{bmatrix} = X \begin{bmatrix}
4 & 3 \\
3 & 2
\end{bmatrix}. $$
6. (9 points total, 3 points each)

(a) Suppose that $A, B, C$ are invertible $n \times n$ matrices. Find, with explanation, the inverse of the matrix $ABC$. Make sure that you clearly and completely justify your response.

(b) Suppose that $A$ is an invertible $4 \times 4$ matrix. How many solutions does the linear system $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ have? Why?
(c) Create a $50 \times 50$ matrix $A$ in the following way. The first row of $A$ consists of all 1’s. The second row of $A$ consists of all 2’s. The third row of $A$ consists of all 3’s. Continue in this way, until the 50-th row of $A$ consists of all 50’s. In general, the $n$-th row of $A$ consists of all $n$’s. Is $A$ invertible? Why or why not? Hint: this problems does not require a significant amount of computation.
7. (5 points) Determine whether each of the following statements is True or False. These questions will be scored as follows: +1 points for a correct answer, 0 points for no response, and \(-1/2\) points for an incorrect answer. No explanation is necessary.

(a) If \(A\) and \(B\) are invertible \(n \times n\) matrices and \(AB = BA\), then \(A^{-1}B = BA^{-1}\).

(b) Every homogeneous linear system has at least one solution.

(c) If \(A\) is a \(5 \times 5\) matrix such that \(\det(A) = 5\), then \(\text{rref}(A)\) must have 5 pivots.

(d) The function \(y = e^{2x}\) is a solution of the differential equation \(y'' + y' - 6y = 0\).

(e) If \(A\) is an invertible \(n \times n\) matrix and if \(r\) is a real number such that \(r \neq 0\), then \((rA)^{-1} = rA^{-1}\).