Math 206 Exam 1
Thursday, October 15, 2009

Name:

- You have 50 minutes to complete this exam.
- No notes, books, calculators, or other references are allowed.
- You must show all work to receive credit. Answers for which no work is shown will receive no credit (unless specifically stated otherwise).
- There are problems on the front and back of each page. Make sure that you answer all of the questions. There is extra paper available if you need it.
- Good luck! Have fun! Eat candy as necessary.

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1. (7 points) For what value(s) of $h$ are the vectors

$$
\begin{bmatrix}
1 \\
2 \\
1
\end{bmatrix},
\begin{bmatrix}
2 \\
5 \\
3
\end{bmatrix},
\begin{bmatrix}
-1 \\
-4 \\
h
\end{bmatrix}
$$

linearly dependent?
2. (8 points) Suppose that $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is a linear transformation such that

\[
T \left( \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \quad T \left( \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \quad T \left( \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}.
\]

Find the standard matrix representation $A$ of $T$. 
3. (8 points total, 4 points each)

(a) Give an example of 5 linearly independent vectors in \( \mathbb{R}^4 \), or explain why it is impossible to do so.

(b) Give an example of 3 vectors that span \( \mathbb{R}^2 \), or explain why it is impossible to do so.
4. (6 points total, 3 points each) Use the following matrices for this problem.

\[ A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 4 & 0 \\ 5 & 0 & -1 & 2 \end{bmatrix}. \]

(a) Compute \( AB \), or state that it is not defined.

(b) Compute \( (BA)^T \), or state that it is not defined.
5. (6 points) Suppose that $A$ is a $3 \times 3$ matrix such that the first two columns of $\text{rref}(A)$ contain pivots and

$$A \begin{bmatrix} 8 \\ -4 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$ 

Find $\text{rref}(A)$. 
6. (7 points) Is it possible to construct a $4 \times 3$ matrix $A$ and a $3 \times 4$ matrix $B$ such that the columns of $AB$ are linearly independent? If so, give an example of two such matrices. If not, explain why not.
7. (8 points) Determine whether each of the following statements is True or False. These questions will be scored as follows: +1 points for a correct answer, +1/2 points for no response, and 0 points for an incorrect answer.

(a) If \( u \) and \( v \) are two solutions of \( A\mathbf{x} = \mathbf{0} \), then any vector in \( \text{Span}(u, v) \) is also a solution of \( A\mathbf{x} = \mathbf{0} \).

(b) If \( A \) and \( B \) are both \( 3 \times 3 \) matrices, then \((AB)^T = A^TB^T\).

(c) If \( A \) and \( B \) are matrices such that \( AB \) and \( BA \) are both defined, then \( A \) and \( B \) must be square matrices.

(d) If \( A \) and \( B \) are matrices such that \( AB \) is defined, and if \( B \) has a column of zeros, then so does \( AB \).

(e) If \( A \) is an \( m \times n \) matrix such that \( \text{rref}(A) \) has \( m \) pivot columns, then the linear transformation \( T(\mathbf{x}) = A\mathbf{x} \) is a one-to-one mapping.

(f) If \( A \) is an \( m \times n \) matrix such that the equation \( A\mathbf{x} = \mathbf{b} \) is consistent for all vectors \( \mathbf{b} \) in \( \mathbb{R}^m \), then \( \text{rref}(A) \) has \( m \) pivot columns.

(g) If \( A \) is an \( m \times n \) matrix such that the equation \( A\mathbf{x} = \mathbf{b} \) is consistent for some \( \mathbf{b} \) in \( \mathbb{R}^m \), then the columns of \( A \) span \( \mathbb{R}^m \).

(h) If \( A \) and \( B \) are \( n \times n \) matrices, then \((A + B)(A - B) = A^2 - B^2\).