Problem 1. Let \( P_0 = (0, y_0) \) and \( P_1 = (0, y_1) \) be two points on the vertical line \( L = \{(0, y) | y > 0\} \subset \mathbb{H}^2 \) and \( y_1 > y_0 \). Let \( g \) be a hyperbolic geodesic passing through \( P_0 \). Show that the followings are equivalent:

1. \( P_0 \) is the closest point to \( P_1 \) on \( g \) (with respect to the hyperbolic distance function on \( \mathbb{H}^2 \)).

2. \( g \) is the complete geodesic orthogonal to \( L \) at \( P_0 \): it is the Euclidean semi-circle centered at \((0, 0)\) going through \((0, y_0)\).

Problem 2. Consider three semicircles \( C, C', C_0 \) that are mutually tangent at points on a line \( L \subset \mathbb{R}^2 \) and \( C', C_0 \) are inside of \( C \). Inscribe a chain of circles \( C_1, \ldots, C_n \) such that each \( C_i \) is tangent to \( C_{i-1}, C \) and \( C' \). Show that the center of \( C_i \) is at a distance \( i \times d_i \) from \( L \), where \( d_i \) is the diameter of \( C_i \).

Problem 3. Let \( C_1 \) be a circle lying within the interior of a second circle \( C_2 \). Suppose that there exists a chain of circles such that each circle is tangent to both \( C_1 \) and \( C_2 \), and such that adjacent circles are tangent. Show that if one such chain exists, then no matter where we start the circle, we will end up with a chain.

Problem 4. Given two distinct points \( z_1, z_2 \in \mathbb{H}^2 \). Let \( x_1, x_2 \in \mathbb{R} \cup \{\infty\} \) be the two end points of the complete geodesic \( g \) passing through \( z_1 \) and \( z_2 \) such that \( x_1, z_1, z_2 \) and \( x_2 \) occur in this order on \( g \). Show that

\[
d_{\text{hyp}}(z_1, z_2) = \log\left(\frac{(z_1 - x_2)(z_2 - x_1)}{(z_1 - x_1)(z_2 - x_2)}\right).
\]

Problem 5. Let \( P = (x, y) \in \mathbb{H}^2 \). Show that the ball \( B_{\text{hyp}}(P, r) \) centered at \( P \) is the Euclidean disc with Euclidean radius \( 2y \sinh(r) \) and with Euclidean center \( (x, 2y \cosh(r)) \).

Problem 6. It is a theorem in Euclidean geometry that the medians of a triangle are concurrent. Does this theorem also hold in \( \mathbb{H}^2 \)?

Problem 7. Assume that \( P \) lies on the hyperbolic circle \( B_{\text{hyp}}(Q, R) \subset \mathbb{H}^2 \), show that:

- There is a unique \( h \)-tangent to \( C \) at \( P \), and
- the tangent line is perpendicular to the \( h \)-line \( PQ \).
Problem 8.

1. The point $A$ lies on the h-circle with diameter $BC$ if and only if $\angle BAC = \angle ABC + \angle ACB$.

2. Assume that for triangles $\triangle_1 = ABC$ and $\triangle_2 = PQR$ in $\mathbb{H}^2$ we have:
   
   - $d_{hyp}(A, B) = d_{hyp}(P, Q)$,
   - $\angle BAC = \angle QPR$, and
   - $d_{hyp}(A, C) = d_{hyp}(P, R)$,

   Show that these triangles are congruent: there exists an isometry $\phi$ of $\mathbb{H}^2$ such that $\phi(\triangle_1) = \triangle_2$.

Problem 9. Prove that if $g_0$ and $g_1$ are complete hyperbolic geodesics in $\mathbb{H}^2$ sharing an end point at infinity (on the line $\{(x, 0) \mid x \in \mathbb{R}\}$), then there does not exist a hyperbolic geodesic perpendicular to both these geodesics.