THIS IS A CLOSED BOOK, CLOSED NOTES EXAM. NO CALCULATORS OR OTHER ELECTRONIC DEVICES ARE PERMITTED.

IF YOU NEED EXTRA SPACE, PLEASE USE THE BACK OF THE PREVIOUS PROBLEM PAGE. SO EXTRA WORK FOR PROBLEM 1 WOULD GO ON THE BACK OF THIS COVER SHEET, ETC.

MOST PROBLEMS ARE WORTH 10 POINTS. PROBLEM TEN IS 12 POINTS AND PROBLEM ELEVEN IS 8 POINTS. THE TOTAL IS 120 POINTS.

THE TERMS $C^1$ OR $C^2$ FOR FUNCTIONS AND VECTOR FIELDS MEANS THAT THEY HAVE CONTINUOUS FIRST PARTIAL DERIVATIVES OR CONTINUOUS FIRST AND SECOND PARTIAL DERIVATIVES, RESPECTIVELY, ON THEIR DOMAINS.

Please sign the following, and make sure your name is legible:

"On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

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1(a)[4]. Sketch the region of integration in the plane for
\[ \int_0^1 \int_{1+x^3}^{1+\sqrt{x}} f(x, y) \, dy \, dx. \]

The points (0, 1) and (1, 2) are connected by the curves \( y = 1 + x^3 \) and \( y = 1 + \sqrt{x} \), and the region is between them. So, \( 0 \leq x \leq 1, \ 1 + x^3 \leq y \leq 1 + \sqrt{x} \), an \( x \)-simple region.

(b)[6]. Reverse the order of integration for the integral in part (a).
\[ \int_1^{\sqrt{y-1}} \int_{(y-1)^2}^{\sqrt{y-1}} f(x, y) \, dx \, dy \]
2(a). Let $W$ be the solid object in space bounded by the $xy$-plane, the cylinder $x^2 + y^2 = 4$, and the paraboloid $z = x^2 + y^2$. Set up AND EVALUATE a triple integral in cylindrical coordinates in the order $\int \int \int - dz \ dr \ d\theta$ that gives the volume of the object.

\[
\int_0^{2\pi} \int_0^2 \int_0^r r \ dz \ dr \ d\theta = \int_0^{2\pi} \int_0^2 r^3 \ dr \ d\theta = 8\pi
\]

(b). Set up but DO NOT EVALUATE an integral in cylindrical coordinates in the order $\int \int \int - dr \ dz \ d\theta$ that gives the volume of the same object from part (a).

\[
\int_0^{2\pi} \int_0^4 \int_{\sqrt{z}}^2 r \ dr \ dz \ d\theta
\]
3(a)[4]. A wire is in the shape of a coil described by \((\cos(2\pi t), \sin(2\pi t), t), \quad 0 \leq t \leq 4.\) Suppose the electric charge density along the wire is given by \(e(x, y, z) = \sqrt{z}.\) Set up an explicit integral that gives the total electric charge on the wire. DO NOT EVALUATE.

The tangent vector is \(\mathbf{X}(t) = (-2\pi \sin(2\pi t), 2\pi \cos(2\pi t), 1),\) with length \(||\mathbf{X}'(t)|| = \sqrt{4\pi^2 + 1}.\) The total charge is
\[
\int e \ ds = \int_0^4 \sqrt{t} \ ||\mathbf{X}'(t)|| \ dt = \int_0^4 \sqrt{t} \ \sqrt{4\pi^2 + 1} \ dt
\]

(b)[6]. Suppose surface \(S\) is the portion of the paraboloid \(x^2 + y^2 = 4z\) with \(1 \leq z \leq 4.\) Parametrize \(S\) by \(\mathbf{r}(s, t) = (2t \cos(s), 2t \sin(s), t^2)\) with \(0 \leq s \leq 2\pi, \quad 1 \leq t \leq 2.\) If the mass density at points of \(S\) is given by \(\delta = z,\) Use the parametrization to express the moment of \(S\) with respect to the plane \(z = 0\) as an explicit integral over a domain in the \(st\)-plane. DO NOT EVALUATE.

First find \(\mathbf{N} = \mathbf{r}_s \times \mathbf{r}_t = (4t^2 \cos(s), 4t^2 \sin(s), -4t)\) with length \(||\mathbf{N}|| = \sqrt{16t^4 + 16t^2}.\) The moment with respect to plane \(z = 0\) is
\[
\int \int_S z \ \delta \ dS = \int_0^{2\pi} \int_1^2 t^2 \ t^2 \ ||\mathbf{N}|| \ dt \ ds = \int_0^{2\pi} \int_1^2 t^2 \ t^2 \ \sqrt{16t^4 + 16t^2} \ dt \ ds
\]
4. Let $D$ be the triangle in the first quadrant of the $xy$-plane bounded by the $x$-axis, the $y$-axis, and the line $x + y = 1$. The transformation $T(u, v) = (x, y) = (1 - u, uv)$ maps the square $0 \leq u \leq 1, \ 0 \leq v \leq 1$ in the $uv$-plane onto the triangle $D$ in the $xy$-plane.

(a)[3]. Find the Jacobian determinant $\frac{\partial (x, y)}{\partial (u, v)}$.

$$\det \begin{pmatrix} -1 & 0 \\ v & u \end{pmatrix} = -u$$

(b)[3]. Fill in the limits of integration asterisks for

$$\int \int_D \left( \sin((1 - x)^2) + \frac{y}{1 - x} \right) \, dA = \int_* ^* \int_* ^* \left( \sin((1 - x)^2) + \frac{y}{1 - x} \right) \, dx \, dy$$

The triangle $D$ is the $y$-simple region $0 \leq y \leq 1, \ 0 \leq x \leq 1 - y$. So the answer is

$$\int_0^1 \int_0^{1 - y} \left( \sin((1 - x)^2) + \frac{y}{1 - x} \right) \, dx \, dy$$

(c)[4]. Use the change of variables theorem to express $\int \int_D \left( \sin((1 - x)^2) + \frac{y}{1 - x} \right) \, dx \, dy$ as an explicit integral $\int \int_{D^*} g(u, v) \, dudv$ over a region $D^*$ in the $uv$-plane. DO NOT EVALUATE. Which looks easier to evaluate, the integral in (b) or (c)?

Since $x = 1 - u$ we have $1 - x = u$. Also $y/1 - x = uv/u = v$. The nasty integral in (b) becomes the doable double integral

$$\int_0^1 \int_0^1 \left( \sin(u^2) + v \right) \frac{\partial (x, y)}{\partial (u, v)} \, dudv = \int_0^1 \int_0^1 \left( \sin(u^2) + v \right) \, u \, dudv$$
5(a)[3]. Find a potential function for the vector field $\vec{F}(x, y, z) = \sqrt{1 + x^2 + y^2 + z^2} (x, y, z)$ on $\mathbb{R}^3$ or give a reason no such potential function exists.

All continuous central radial vector fields are conservative. A potential function here is

$$f(x, y, z) = \frac{1}{3} (1 + x^2 + y^2 + z^2)^{3/2}$$

(b)[3]. Is $\vec{G} = 2xy \vec{i} + (x^2 - y) \vec{j}$ a conservative vector field on $\mathbb{R}^2$? Why or why not?

Sure, you can check $Q_x = P_y = 2x$ on the entire (simply-connected) plane $\mathbb{R}^2$, or you can just write down a potential function $g(x, y) = x^2y - y^2/2$

(c)[4]. Find the work done by the vector field $\vec{G}$ of part (b) on a particle that moves from $(-1, -1)$ to $(1, 1)$ clockwise around the ellipse $2x^2 + 3y^2 = 5$.

We use the potential function $g(x, y) = x^2y - y^2/2$ from part (b). The work is

$$g(1, 1) - g(-1, -1) = \frac{1}{2} - \frac{-3}{2} = 2$$
6. Let $S$ be the portion of the sphere $x^2 + y^2 + z^2 = 25$ that lies on or below the plane $z = 4$. (So, $S$ is most of the sphere.) Let $\vec{F} = (x + 3y^2 + z^2) \vec{i} + (x^2 + y - z^2) \vec{j} + (xy - 2z) \vec{k}$.

(a) Calculate $\text{div}(\vec{F})$.

For $\vec{F} = (P, Q, R)$, we have $\text{div}(\vec{F}) = P_x + Q_y + R_z = 1 + 1 - 2 = 0$ in our question.

(b) Compute the flux $\iint_S \vec{F} \cdot \vec{n} \, dS$, where $\vec{n}$ is the unit normal given by $\vec{n} = \frac{1}{5}(x, y, z)$ at points of $S$. Explain your work clearly. For your own protection, NO CREDIT will be given for attempts that try to parametrize $S$.

[Hint: $S$ is not a closed surface. But $S$ together with some other surface does form a closed surface. Use part (a). You can also use symmetry and knowledge of certain areas to evaluate certain integrals without actually integrating.]

We get a closed surface if we add to $S$ the flat disk $D$ given by $x^2 + y^2 \leq 9$, $z = 4$. Since $\text{div}(\vec{F}) = 0$ everywhere in $\mathbb{R}^3$, Gauss’s theorem implies

$$0 = \iiint_{S \cup D} \vec{F} \cdot \vec{n} \, dS = \iint_S \vec{F} \cdot \vec{n} \, dS + \iint_D \vec{F} \cdot \vec{n} \, dS.$$

The normal vector $\frac{1}{5}(x, y, z)$ on $S$ is the outward normal to the closed surface, so on disk $D$ we have $\vec{n} = \vec{k}$. Referring to the formula for $\vec{F}$, and using $z = 4$ on $D$, we now have

$$\iint_S \vec{F} \cdot \vec{n} \, dS = -\iint_D (xy - 2z) \, dS = \iint_D (8 - xy) \, dS = 8(\text{Area of } D) = 72\pi,$$

since the integral of $xy$ over the disk $D$ is 0 by symmetry.
7. In this problem you will use the graph parametrization of a hemisphere given by
\[ T(x, y) = (x, y, \sqrt{a^2 - x^2 - y^2}), \text{ for } (x, y) \text{ in the disk } D \text{ described by } x^2 + y^2 \leq a^2 \text{ in the } xy\text{-plane.} \]

(a) [3]. Give the standard normal vector \( \vec{N} = \vec{T}_x \times \vec{T}_y \) for this parametrization and find the length \( ||\vec{N}||. \) [You can just give the answers if you remember, otherwise work it out.]

For a graph \( z = g(x, y) \) the formula is \( \vec{N} = (-g_x, -g_y, 1). \) In our case \( g(x, y) = \sqrt{a^2 - x^2 - y^2}, \) so this becomes
\[
\vec{N} = \left( \frac{x}{\sqrt{a^2 - x^2 - y^2}}, \frac{y}{\sqrt{a^2 - x^2 - y^2}}, 1 \right)
\]
with length
\[
||\vec{N}|| = \sqrt{\frac{x^2 + y^2 + (a^2 - x^2 - y^2)}{a^2 - x^2 - y^2}} = \frac{a}{\sqrt{a^2 - x^2 - y^2}}
\]

(b) [3]. The surface area of \( S \) is given by the integral \( \iint_D ||\vec{N}|| \ dA. \) Is this integral an improper integral or a proper integral? Give a reason for your answer.

Improper integral since the integrand \( ||\vec{N}|| \) becomes infinite on the boundary of \( D, \) where \( a^2 - x^2 - y^2 = 0. \)

(c) [4]. Showing your work, EVALUATE the integral \( \iint_D ||\vec{N}|| \ dA. \)
[You know the answer, but you must show work here.]

We use polar coordinates to integrate over \( D, \) but we should do the improper integral properly (chuckle, chuckle...)
\[
\lim_{b \to a} \int_{0}^{2\pi} \int_{0}^{b} \frac{ar}{\sqrt{a^2 - r^2}} \ dr \ d\theta = \lim_{b \to a} \left( \frac{2\pi}{2} \right) = 2\pi a^2
\]
8. In this problem, we consider the plane vector field
\[ \vec{F} = (P(x, y), Q(x, y)) = \left( \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} + 2x \right) \]
which satisfies \( \partial Q/\partial x - \partial P/\partial y = 2 \) at all points where \( \vec{F} \) is defined.

(a) If \( C_1 \) is the circle \( x^2 + y^2 = 1 \) oriented counterclockwise, calculate \( \oint_{C_1} \vec{F} \cdot d\vec{s} \).

Green’s theorem does not apply directly because \( Q_x \) and \( P_y \) are not continuous, or even defined, at the point \((0, 0)\) inside the circle. We can write
\[ \vec{F} = \vec{F}_1 + \vec{F}_2 = \left( \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right) + (0, 2x). \]

On circle \( C_1 \) oriented counterclockwise, we have \( x^2 + y^2 = 1 \) and the unit tangent vector is \( \vec{t} = (-y, x) \). So
\[ \int_{C_1} \vec{F}_1 \cdot d\vec{s} = \int_{C_1} \vec{F}_1 \cdot \vec{t} \, ds = \int_{C_1} 1 \, ds = \text{length } C_1 = 2\pi. \]

For \( \vec{F}_2 \), we CAN use Green’s theorem. Call \( D \) the disk bounded by \( C_1 \). Then
\[ \int_{C_1} \vec{F}_2 \cdot d\vec{s} = \int_{C_1} 0 \, dx + 2 \, x \, dy = \int_D 2 \, dxdy = 2 \text{ area } D = 2\pi. \]

The final answer is \( 2\pi + 2\pi = 4\pi \). This solution is pretty tricky. It is also easy enough to parametrize \( C_1 \) as \((\cos(t), \sin(t)), \quad 0 \leq t \leq 2\pi \) with tangent \((-\sin(t), \cos(t))\). One gets quickly to the integral
\[ \int_{C_1} \vec{F} \cdot d\vec{s} = \int_0^{2\pi} 1 + 2\cos^2(t) \, dt = 2\pi + 2\pi = 4\pi \]

(b) If \( C_2 \) is the ellipse \( \frac{x^2}{25} + \frac{y^2}{9} = 1 \) oriented counterclockwise, calculate \( \oint_{C_2} \vec{F} \cdot d\vec{s} \).

[You may use the formula for the area inside an ellipse, but NO CREDIT for attempts that parametrize the ellipse. Remember \( \partial Q/\partial x - \partial P/\partial y = 2 \) at all points where \( \vec{F} \) is defined.]

We can apply Green to the region \( R \) BETWEEN curves \( C_1 \) and \( C_2 \). The result with both counterclockwise orientations is
\[ \oint_{C_2} \vec{F} \cdot d\vec{s} = \oint_{C_1} \vec{F} \cdot d\vec{s} + \iint_R 2 \, dA = 4\pi + 2(\text{area } R) = 4\pi + 2(15\pi - \pi) = 4\pi + 28\pi = 32\pi \]
9[10]. Write T (True) or F (False) in the margin to the left of each statement. There are ten statements, continuing on the next page. One point for each correct answer.

(i). If \( f(x) \) and \( g(y) \) are two continuous functions of one variable then the double integral of the product \( f(x)g(y) \) over an \( x \)-simple region \( a \leq x \leq b, \ p(x) \leq y \leq q(x) \) can always be computed as the product \( (\int_{a}^{b} f(x) \, dx)(\int_{p(x)}^{q(x)} g(y) \, dy) \).

F. Such a formula only makes sense for rectangles \( a \leq x \leq b, \ c \leq y \leq d \).

(ii). The region in the plane bounded by the two circles \( x^2 + y^2 = 4 \) and \( x^2 + (y - 1)^2 = 1 \) is neither an \( x \)-simple nor a \( y \)-simple region.

T. Best seen by drawing the two circles, one of which is inside the other.

(iii). If \( S \) is a surface with boundary in space and if \( S \) has constant density, then the \( y \) coordinate of the center of mass of \( S \) is the average value of the function \( y \) on \( S \).

T. This is seen by just comparing the definitions of average value and center of mass coordinate.

(iv). If \( \bar{T}(u, v) = (x(u, v), y(u, v), z(u, v)) \) is a linear transformation mapping \( R^2 \) onto a plane in \( R^3 \) and if \( \vec{N} = \bar{T}_u \times \bar{T}_v \) is the standard normal vector to \( \bar{T}(R^2) \), then for any region \( D \) in the \( uv \)-plane that has an area, the area of \( \bar{T}(D) \) is \( ||\vec{N}|| \text{area}(D) \).

T. It is certainly true for little rectangles in the \( uv \)-plane, which transform to parallelograms. But then since \( \bar{T} \) is linear, all areas get multiplied by the same factor.

(v). \( \vec{\nabla} \times (\vec{\nabla} f) = \vec{0} \) for all \( C^2 \) functions \( f(x, y, z) \) on \( R^3 \).

T. This is a standard identity, \( \text{Curl Grad}(f) = \vec{0} \)
(vi). If $f(x, y, z)$ is a $C^2$ function on $R^3$ then $\text{Div} \ \text{Grad}(f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$.

T. This is also a standard identity.

(vii). If $\vec{F} = (P, Q)$ is a $C^1$ vector field on the domain $W = R^2 - (0, 0)$, [that is, $W$ is $R^2$ with the origin removed], and if $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$ on domain $W$, then $\oint_C P \, dx + Q \, dy = 0$ for all simple closed curves $C$ in $W$.

F. We have emphasized examples where Green’s theorem might not apply because $Q_x$ and $P_y$ are not continuous at $(0,0)$. See the example $\vec{F}_1$ in Problem 8.

(viii). If $S$ is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$ in $R^3$ with upward pointing unit normal $\vec{n}$ and if $\vec{F} = (-yz, y^2, z)$ then $\int_S \vec{F} \cdot \vec{n} \, dS < 0$.

F. We have $\vec{n} = \frac{1}{2}(x, y, z)$, so the integral is $\frac{1}{2} \int_S -xyz - y^3 + z^2 \, dS$. The first two terms give 0 by symmetry. The last term gives a positive number.

(ix). On every smooth surface with or without boundary in $R^3$ there are two continuous unit normal vector fields.

F. The Mobius strip is a counterexample, with no continuous unit normal vector fields.

(x). If $f(u)$ is any continuous function for $u > 0$, and if $r = \sqrt{x^2 + y^2 + z^2}$, then the vector field $f(r)(x, y, z)$ is conservative in the region $R^3 - (0,0,0)$.

T. All continuous radial central vector fields are conservative. Here, if $g'(u) = uf(u)$ then $f(r)(x, y, z) = \text{Grad}(g(r))$, since

$$\frac{\partial}{\partial x} g(\sqrt{x^2 + y^2 + z^2}) = \frac{g'(r)}{r} x = f(r)x,$$

and similarly for the partials with respect to $y$ and $z$. 


11
For each of the regions $W$ in $\mathbb{R}^3$ described below, suppose $\vec{F}$ is a $C^1$ vector field with $\text{Curl } \vec{F} = \vec{0}$ throughout $W$. In each case, decide if you can you be certain that $\vec{F}$ is a conservative vector field throughout $W$? Answer YES or NO. There are six regions. One point for each correct answer.

If every simple closed curve in region $W$ bounds a surface in $W$ (that is, if $W$ is simply connected) then $\text{Curl}(\vec{F}) = \vec{0}$ will guarantee by Stokes theorem that $\vec{F}$ has path independent line integrals in $W$, hence is conservative in $W$.

(i). $\mathbb{R}^3$ with the $x$ axis removed. NO

(ii). $\mathbb{R}^3$ with the circle $x^2 + y^2 = 1$ in the $xy$-plane removed. NO

(iii). $\mathbb{R}^3$ with the origin removed. YES

(iv). $\mathbb{R}^3$ with the plane $z = 0$ removed. YES

(v). The octant $x > 0, y > 0, z > 0$ in $\mathbb{R}^3$. YES

(vi). The region in $\mathbb{R}^3$ described as all $(x, y, z)$ with $4 < x^2 + y^2 + z^2 < 25$. YES

(b)[6]. Write T(true) or F(false) in the margin to the left of each of the six statement below. One point for each correct answer.

(i). If $\vec{F}$ is a $C^1$ vector field on all of $\mathbb{R}^2$ with $\text{Div } \vec{F} = 0$ then the flux of $\vec{F}$ across any simple closed curve $C$ in $\mathbb{R}^2$ is always zero. (TRUE. BY 2 DIM’L DIVERGENCE THEOREM)

(ii). There exists a vector field $\vec{G}$ on $\mathbb{R}^3$ with $\text{Div}(\vec{G}) = y^2x^2$. (TRUE. TAKE $\vec{G} = (0, 0, \frac{1}{2}z^3y^2x^2)$)

(iii). If $\vec{F}$ is a $C^1$ vector field on all of $\mathbb{R}^3$ and if $\vec{\nabla} \cdot \vec{F} = 1$ at all points of a solid region $W$ in $\mathbb{R}^3$ and if $S$ is the boundary of $W$ then the outward flux $\int_S \vec{F} \cdot \vec{n} \, dS$ always equals the volume of $W$. (TRUE. BY GAUSS DIVERGENCE THEOREM)
(iv). If $\vec{F}$ is a $C^1$ vector field on all of $\mathbb{R}^3$ and if $\int_C \vec{F} \cdot \vec{d}s = 0$ for all simple closed curves $C$ in $\mathbb{R}^3$, then $\vec{F} = \vec{0}$.

(FALSE. TAKE ANY GRADIENT VECTOR FIELD)

(v). If $\vec{F}$ is a $C^2$ vector field on $\mathbb{R}^3 - (0,0,0)$, that is, everywhere except the origin, and if $S$ is the sphere of radius 1 and center $(0,0,0)$, then $\int_S \text{Curl}(\vec{F}) \cdot \vec{d}S = 0$.

(TRUE. THIS IS A CONSEQUENCE OF STOKES THEOREM, IN PARTICULAR "SURFACE INDEPENDENCE" OF CURL VECTOR FIELD INTEGRALS)

(vi). If $\vec{F}$ is a conservative vector field, $C^1$ on all of $\mathbb{R}^3$, and if $S$ is a closed oriented surface, that is, with no boundary, then $\int_S \vec{F} \cdot \vec{d}S = 0$.

(FALSE. GAUSS DIVERGENCE THEOREM SAYS THIS INTEGRAL IS $\pm$ THE TRIPLE INTEGRAL OVER THE SOLID OF SOME $\text{Div}(\text{Grad}(f)) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$, WHICH CAN BE ANYTHING)
11. Consider the vector field on $\mathbb{R}^3 - (0,0,0)$ given by $\vec{F} = \frac{1}{r^3}(x,y,z)$ where $r = \sqrt{x^2 + y^2 + z^2}$. Let $S_a$ be the sphere of radius $a$ given by $x^2 + y^2 + z^2 = a^2$ with outward unit normal $\vec{n}$.

(a) [4]. Calculate the flux integral $\int \int_{S_a} \vec{F} \cdot \vec{n} \ dS$, WITHOUT using any parametrization. SHOW WORK. [You may assume the surface area of $S_a$ is $4\pi a^2$.]

The normal is $\vec{n} = \frac{1}{a}(x,y,z)$. On the sphere, $\vec{F} = \frac{1}{a^3}(x,y,z)$. So the integral is

$$\int \int_{S_a} \vec{F} \cdot \vec{n} \ dS = \int \int_{S} \frac{a^2}{a^4} \ dS = \frac{1}{a^2} \text{area}(S_a) = 4\pi$$

(b) [2]. Routine derivative calculations yield $\text{Div}(\vec{F}) = 0$ on the domain of $\vec{F}$. Why doesn’t Gauss’s theorem give the result $\int \int_{S_a} \vec{F} \cdot \vec{n} \ dS = 0$?

Gauss’ theorem does not apply because the vector field and its partial derivatives are not continuous throughout the solid ball bounded by $S_a$.

(c) [2]. Is it possible that $\vec{F} = \text{Curl}(\vec{G})$ for some vector field $\vec{G}$ on $\mathbb{R}^3 - (0,0,0)$? Give a reason why or why not.

No. A consequence of Stokes theorem is that the integral of a Curl over any closed surface is 0. Since part (a) shows $\vec{F}$ does have non-0 integrals over closed surfaces, it cannot possibly be a Curl.
12[10]. The graph on the next two pages ‘plots’ a vector field \( \vec{F}(x, y) = (P(x, y), Q(x, y)) \), oriented curves \( A, B, C \), and points \( M, N, R, T \). One assumes \( \vec{F} \) is a \( C^1 \) vector field. Answer the following ten questions. There are two copies of the graph so you can tear off the last page while contemplating the questions. One point for each correct answer.

(i). Is the circulation of \( \vec{F} \) around curve \( A \) positive, negative, or 0? POSITIVE

(ii). If \( \vec{n} \) is the continuous unit normal to curve \( C \) pointing downward (except near \( N \)), is the flux across \( C \) in the \( \vec{n} \) direction positive, negative, or 0? POSITIVE

(iii). Is the quantity \( \int_C \vec{F} \cdot d\vec{s} \) positive, negative, or 0? POSITIVE

(iv). Is the value of \( \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \) at \( T \) most likely positive, negative, or 0? POSITIVE

(v). Is the quantity \( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \) at \( M \) most likely positive, negative, or 0? NEGATIVE

(vi). Is the divergence of \( \vec{F} \) at \( N \) most likely positive, negative, or 0? NEGATIVE

(vii). If \( D \) is the plane region bounded by curve \( A \), is the quantity \( \iint_D \text{Div}(\vec{F}) \, dxdy \) positive, negative, or 0? NEGATIVE

(viii). If \( \vec{F} \) is a fluid flow velocity field, would a small paddle at \( N \) most likely spin clockwise, counterclockwise, or not at all? COUNTERCLOCKWISE

(ix). If \( \vec{F} \) is a force field, is the work done by \( \vec{F} \) on a particle moving around directed curve \( B \) positive, negative, or 0? NEGATIVE

(x). Is it possible that \( \vec{F} = \nabla f \) for some function \( f(x, y) \)? NO, SEE (i) AND (ix)