Problem 1.

a) (10 points) Sketch the region of integration of \( \int_0^2 \int_y^2 \cos(x^2) \, dx \, dy \)

b) (10 points) Evaluate \( \int_0^2 \int_y^2 \cos(x^2) \, dx \, dy \)

Solution: After the change of order of integration we get:
\[
\int_0^2 \int_0^x \cos(x^2) \, dy \, dx = \int_0^2 x \cdot \cos(x^2) \, dx = \frac{1}{2} \sin(x^2) \bigg|_0^2 = \frac{\sin 4}{2}
\]

Problem 2. (20 points) Let \( W \) be the solid defined by the inequalities:
\[
\begin{cases}
  z \leq 1 - y^2 \\
  z \geq y^2 - 1 \\
  x + z \leq 1 \\
  x \geq 0
\end{cases}
\]

For the above solid \( W \) write \( \iiint_W f(x, y, z) \, dV \) as a triple iterated integral over an \( x \)-simple region. (there are still two possible choices of the order of the integrals, so just choose one of them...)

Solution:
\[
\int_{-1}^1 \int_{y^2-1}^{1-y^2} \int_0^{1-z} f(x, y, z) \, dx \, dz \, dy
\]

Problem 3. (20 points) Find the moment of inertia with respect to the line \( x = 1 \) of a thin sheet of constant density \( \rho = 1 \) bounded by the curve \( y = \left( \sin^2 x \right) / (x - 1)^2 \), the \( x \)-axis and the strip \( \pi \leq x \leq 2\pi \).
You may use the formula:
\[ \int_{\pi}^{2\pi} \sin^2 x \, dx = \frac{\pi}{2} \]

Solution:
\[ I_{x=1} = \int_{\pi}^{2\pi} \frac{\sin^2 x/(x-1)^2}{(x-1)^2} \, dx = \int_{\pi}^{2\pi} \sin^2 x \, dx = \frac{\pi}{2} \]

Problem 4. (20 points) Consider the change of variables \( u = xy, \ v = yz, \ w = xz \).

a) Find the Jacobian \( \frac{\partial(x, y, z)}{\partial(u, v, w)} \).

Hint: Use the fact that \( uvw = x^2 y^2 z^2 \). Solution:
\[ \frac{\partial(u, v, w)}{\partial(x, y, z)} = \begin{vmatrix} y & x & 0 \\ 0 & z & y \\ z & 0 & x \end{vmatrix} = 2xyz \]

so \( \frac{\partial(x, y, z)}{\partial(u, v, w)} = \frac{1}{2xyz} = \frac{1}{2\sqrt{uvw}} \)

b) Find the volume of the region in the first octant enclosed by the hyperbolic cylinders \( xy = 1, \ xy = 4, \ xz = 1, \ xz = 4, \ yz = 4, \ yz = 9 \). Solution:
With the change of variables described in a):
\[ V = \int_{1}^{4} \int_{1}^{4} \int_{1}^{4} \frac{1}{2\sqrt{uvw}} \, dw \, dv \, du = \frac{1}{2} 2\sqrt{u}|_{1}^{4} \cdot 2\sqrt{v}|_{1}^{4} \cdot 2\sqrt{w}|_{1}^{4} = 4 \]

Problem 5. (20 points) Evaluate the integral:
\[ \int_{-a}^{a} \int_{0}^{\sqrt{a^2-y^2}} e^{x^2+y^2} \, dx \, dy \]

Hint: Use polar coordinates. Solution: In polar coordinates:
\[ \int_{-\pi/2}^{\pi/2} \int_{0}^{a} e^{r^2} \, dr \, d\theta = \pi \int_{0}^{a} e^{r^2} \, dr = \frac{\pi}{2} \left( e^a - 1 \right) \]