Math 115 Practice Midterm

October 21, 2012

(i) Please complete the following definition: Let \( s_n \) be bounded sequence. Then \( \limsup s_n \) is defined as follows:

(ii) Show that any ordered field has infinitely many elements. Hint: Show that \( 0, 1, 2 = 1 + 1, 3 = 2 + 1, \ldots \) are all non-equal elements of the field.

(iii) Does the equation \( 3x^3 + 2x^2 + 3x + 2 = 0 \) have a rational solution?

(iv) Let the sequence \( s_n \) be defined by \( s_{n+1} = \sqrt{s_n}, s_1 = s > 1 \). Show \( s_n \) is monotone and bounded. Does \( \lim s_n \) exist? Why or why not? If it exists, compute it, if not show that it does not exist.

(v) Does \( \lim_{n \to \infty} n^{1/n} \) exist? If no, why not, if yes, what is it?

(vi) Let \( s_n \) be a bounded sequence. Show that \( \limsup |s_n| \geq \limsup s_n \).

(vii) (a) Give an example of a sequence of real numbers such that \( -\infty < \inf s_n < \lim s_n < \sup s_n < \infty \).

(b) Give an example of a sequence of real numbers such that \( \liminf s_n = -\infty \) and \( \limsup s_n = \sqrt{2} \).