The midterm on November 18 covers sections 13–17, 21, 23, and 25 of Levandosky (of course, you may need concepts that were covered on the first midterm as well!), and sections 12.1 and 13.1–13.5 of Edwards & Penney. Sample midterms can be found at http://math.stanford.edu/~byoung/math51_fall/solutions.html.

1. Definitions to know
   - linear transformation and the associated matrix, image, preimage, kernel
   - composition $g \circ f$, matrix product
   - one-to-one, onto, invertible, identity matrix $I_n$, inverse $A^{-1}$ of a matrix
   - standard basis for $\mathbb{R}^n$, standard coordinates, change of basis matrix
   - eigenvalue, eigenvector, characteristic polynomial, eigenbasis, diagonalizable
   - symmetric matrix, orthonormal eigenbasis
   - position vector $\vec{r}(t)$, vector-valued function, velocity vector $\vec{v}(t) = \vec{r}'(t)$, acceleration vector $\vec{a}(t) = \vec{r}''(t)$, speed $v(t) = \|\vec{v}(t)\|$, scalar acceleration $a(t) = \|\vec{a}(t)\|
   - graph of a function $f(x, y)$, domain, level curves, contour curves
   - partial derivatives $f_x = \frac{\partial f}{\partial x}, f_y = \frac{\partial f}{\partial y}$, and higher order partials like $f_{xx}, f_{xy}, f_{yy}$
   - global and local maxima and minima, critical points for $f(x, y)$

2. You should know how to...
   - find the matrix for a linear transformation given what it does to the standard basis (or some other basis!)
   - write down the matrices for rotations, projections, and reflections in the plane:
     
     $$\text{Rot}_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix};$$

     $$\text{Proj}_L = \begin{bmatrix} u_2^2 & u_1u_2 \\ u_1u_2 & u_2^2 \end{bmatrix}$$

     if $\vec{u} = [u_1 \ u_2]$ is a unit vector in the direction of the line $L$; and

     $$\text{Ref}_L = 2\text{Proj}_L - I_{\mathbb{R}^2}$$

   - multiply two matrices
   - find the inverse of an invertible matrix using $\text{rref}[A|I_n] = [I_n|A^{-1}]$
   - find the inverse of a $2 \times 2$ matrix using $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
   - calculate the determinant of a matrix, using row or column expansion, or using row reduction to an upper triangular matrix
   - translate between coordinate systems: given $\vec{v}$, find $[\vec{v}]_B$; given $[\vec{v}]_B$, find $\vec{v}$ (use change-of-basis matrix $C$ or other means)
• find the matrix of a linear transformation with respect to \( B \), starting with the standard matrix for the transformation, using \( B = C^{-1}AC \)
• find the matrix of a linear transformation with respect to \( B \), given what the linear transformation does to the vectors in the basis, and use this information and \( A = CBC^{-1} \) to find the standard matrix for the transformation
• find eigenvalues and the corresponding eigenvectors for a matrix (remember how to calculate null spaces!)
• differentiate and integrate a vector-valued function; given either the position vector, velocity vector, or acceleration vector, find the other two
• determine the domain of a function \( f(x, y) \) and where it’s continuous (usually, the same as where it’s defined)
• calculate \( \lim_{(x,y)\to(a,b)} f(x, y) \) using continuity; if this doesn’t work, calculate the limit or show it doesn’t exist, using polar coordinate substitution
• calculate partial derivatives
• find an equation for the tangent plane to the graph of \( f(x, y) \) at the point \((a, b, f(a, b))\), using partial derivatives: 
  \[
  -f_x(a, b)(x - a) - f_y(a, b)(y - b) + (z - f(a, b)) = 0
  \]
• find the global maximum and minimum for a function \( f(x, y) \) over some region \( R \)

3. **You should be familiar with properties and results...**

• what properties define a linear transformation
• the image/preimage of a subspace under a linear transformation is another subspace (the same is true for line segments)
• if \( T : \mathbb{R}^n \to \mathbb{R}^m \) with matrix \( A \), then \( \text{im}(T) = T(\mathbb{R}^n) = C(A) \) and \( \ker(T) = T^{-1}(\{0\}) = N(A) \)
• \( T : \mathbb{R}^n \to \mathbb{R}^m \) is one-to-one precisely when its matrix has rank \( m \), and onto precisely when its matrix has rank \( n \)
• a linear transformation is invertible if and only if its matrix has \( \text{rref} = I_n \); a matrix \( A \) is invertible if and only if \( \det(A) \neq 0 \)
• linearity properties of the determinant (Proposition 17.2) and \( \det(AB) = \det(A) \det(B) \)
• if \( R \) is a region in \( \mathbb{R}^2 \) and \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) is a linear transformation with matrix \( A \), then the area of \( T(R) \) is \( |\det(A)| \) times the area of \( R \); similarly, if \( R \) is a region in \( \mathbb{R}^3 \) and \( T : \mathbb{R}^3 \to \mathbb{R}^3 \) is a linear transformation with matrix \( A \), then the volume of \( T(R) \) is \( |\det(A)| \) times the volume of \( R \)
• if an \( n \times n \) matrix \( A \) has \( n \) distinct eigenvalues, then it is diagonalizable
• if a matrix is symmetric, then it is diagonalizable and has an orthonormal eigenbasis (Spectral Theorem)
• properties of the derivative of a vector-valued function (especially \( \frac{d}{dt}(\vec{u}(t) \cdot \vec{v}(t)) = \vec{u}'(t) \cdot \vec{v}(t) + \vec{u}(t) \cdot \vec{v}'(t) \))

This review sheet can also be found at [http://math.stanford.edu/~lng/math51/mid2.pdf](http://math.stanford.edu/~lng/math51/mid2.pdf). Good luck!