MATH 155: PROBLEM SET 2

Due April 19

1. Let \( d_x \) denote the least common multiple of the integers not exceeding \( x \). Show that
\[
\binom{2n}{n} = \prod_{k=1}^{\infty} d_{2n/k}^{(-1)^{k-1}}.
\]
What can you say about the size of \( d_x \)?

2. Prove directly the identity from class
\[
\mu(n) \log n = -\sum_{ab=n} \mu(a) \Lambda(b).
\]

3. Let \( f \) and \( g \) be two multiplicative functions. Show that \( f*g \) is also multiplicative. A function is called completely multiplicative if it satisfies \( f(mn) = f(m)f(n) \) for all \( m \) and \( n \). Is the convolution of two completely multiplicative functions necessarily completely multiplicative?

4. Prove that
\[
2^{\omega(n)} = \sum_{d^2 m = n} \mu(d)d(m).
\]
(Hint: One way to check such identities is to prove them for prime powers, and then use multiplicativity.) Prove that
\[
\sum_{n \leq x} 2^{\omega(n)} = \frac{6}{\pi^2} x \log x + cx + O(\sqrt{x} \log x),
\]
for some constant \( c \). You may use without proof that \( \zeta(2) = \pi^2 / 6 \).