9.1. \( a \) \( \lim \left( \frac{n+1}{n} \right) = \lim \left( 1 + \frac{1}{n} \right) = \lim (1) + \lim \left( \frac{1}{n} \right) \)  
\( (9.3) \) 
\( = 1 + \lim \left( \frac{1}{n} \right) = 1 \)  
\( (9.76) \)

\( b \) \( \lim \left( \frac{3n+7}{6n-5} \right) = \lim \left( \frac{3+\frac{7}{n}}{6-\frac{5}{n}} \right) \)

\( \lim \left( 3 + \frac{7}{n} \right) = \lim (3) + \lim \left( \frac{7}{n} \right) = 3 + \lim (\frac{7}{n}) = 3 \)  
\( (9.3) \)

and likewise \( \lim \left( 6 - \frac{5}{n} \right) = 6 \), so by 9.6,

\( \lim \left( \frac{3+7/n}{6-5/n} \right) = \frac{3}{6} = \frac{1}{2} \)

\( c \) \( \lim \left( \frac{17n^5 + 73n^4 - 18n^3 + 3}{23n^2 + 13n^3} \right) \)

\( = \lim \left( \frac{17 + 73/n - 18/n^3 + 3/n^5}{23 + 13/n^2} \right) \)

\( \lim \left( 17 + 73/n - 18/n^3 + 3/n^5 \right) \)

\( = \lim (17) + \lim \left( \frac{73}{n} \right) - \lim \left( \frac{18}{n^3} \right) + \lim \left( \frac{3}{n^5} \right) \)  
\( (9.3) \)

\( = 17 + 0 - 0 + 0 = 17 \), and similarly

\( (9.76) \)

\( \lim \left( 23 + \frac{13}{n^2} \right) = 23 \), so by 9.6,

\( \lim \left( \frac{17 + 73/n - 18/n^3 + 3/n^5}{23 + 13/n^2} \right) = \frac{17}{23} \).
9.3. Let \( t_n = a_n^2 + 4a_n, \quad u_n = b_n^2 + 1. \)
\[
\lim t_n = \lim a_n^2 + \lim 4a_n = (\lim a_n)^2 + 4 \lim a_n = a^2 + 4a.
\]
\[
\lim u_n = \lim b_n^2 + \lim 1 = (\lim b_n)^2 + 1 = b^2 + 1.
\]
As \( s_n = t_n / u_n, \) 9.6 gives \( \lim s_n = \frac{a^2 + 4a}{b^2 + 1}. \)

9.11. (a) Let \( b = \inf \{ t_n \} \), then \( t_n > b \) for all \( n \). Given \( M > 0 \), there is \( N \in \mathbb{N} \) such that if \( n > N \) then \( s_n > M - b \). Thus \( s_n + t_n > s_n + b > M - b + b = M \). This proves that \( \lim (s_n + t_n) = +\infty \).

(b) Thm 9.1 says convergent sequences are bounded, so this is a special case of (a).

(c) Since \( \{ t_n \} \) is bounded, in particular it has a greatest lower bound, so \( \inf \{ t_n \} > -\infty \). We then proceed as in (a).

10.1. Nondecreasing: \( c \) only.
Nonincreasing: \( a, f \).
Bounded: \( a, b, d, f \).

10.6. (a) Fix \( \varepsilon > 0 \) and let \( N > -\log_2 (\varepsilon) = \log_2 (\frac{1}{\varepsilon}) \), so \( 2^{-N} < \varepsilon \). Suppose \( m, n > N \). Without loss of generality, assume \( m < n \). Then
\[
|s_n - s_m| \leq \sum_{k=m}^{n-1} |s_{k+1} - s_k| < \sum_{k=m}^{n-1} 2^{-k} < \sum_{k=m}^{\infty} 2^{-k} = 2^{-m+1}.
\]
(Triangle ineq.) (Geom. series)
Since \( m > N \), we have \( m - 1 > N \), so \( 2^{-m+1} \leq 2^{-N} < \varepsilon \).

Thus \( |s_n - s_m| < \varepsilon \), and \( \{s_n\} \) is a Cauchy sequence (hence convergent by Thm. 10.1).

1. No. For a counterexample, consider \( s_n = \sum_{k=1}^{n} \frac{1}{k} \), so

\[
s_1 = 1, \quad s_2 = 1 + \frac{1}{2}, \quad s_3 = 1 + \frac{1}{2} + \frac{1}{3}, \quad \text{etc.}
\]

By construction, \( |s_n - s_m| = \frac{1}{m+1} < \frac{1}{n} \), but \( \lim s_n = \sum_{k=1}^{\infty} \frac{1}{k} \)

diverges to \( +\infty \). (You don’t need to prove this last fact rigorously, at least not at this point in the course.)

10.11. \( t_1 = 1, \quad t_{n+1} = \left( 1 - \frac{1}{n} \right) t_n \) for \( n > 1 \).

2. For \( n > 1 \), \( 0 < 1 - \frac{1}{n^2} < 1 \). Hence if \( t_n \) is positive, we get \( 0 < t_{n+1} < t_n \). By induction, \( \{t_n\} \) is a monotone decreasing sequence, bounded below and above by \( 0 \) and \( 1 \) respectively. Thus it converges by Thm. 10.2.

3. The limit is a positive number less than 1. To my knowledge, the only exact formula is an infinite product involving values of the Riemann zeta function. (You don’t have to know what this is!) The first term of the product is

\[
\frac{1}{\zeta(1)} \approx 0.663 \ ; \text{ the limit should be a bit less.}
\]

11.3. \( \{s_n\} = \{ \cos \left( \frac{a\pi n}{3} \right) \} = \left\{ \frac{3}{2}, -\frac{3}{4}, -1, \frac{3}{4}, \frac{3}{2}, \frac{3}{4}, -1, \ldots \right\} \).

Subsequential limits: \( \{1, \frac{3}{2}, -\frac{3}{4}, -1\} \)

\( \lim \sup = 1, \quad \lim \inf = -1 \).

Neither converges nor diverges to either \( +\infty \) or \( -\infty \).

\( \{t_n\} = \left\{ \frac{3}{4n+1} \right\} = \left\{ \frac{3}{5}, \frac{3}{9}, \frac{3}{13}, \ldots \right\} \).

Subsequential limits: just 0.

\( \lim \sup = 0, \quad \lim \inf = 0 \).

Converges to \( 0 \).
\[ \{ u_n \} = \{ (-\frac{1}{2})^n \} = \{ -\frac{1}{2}, -\frac{1}{4}, -\frac{1}{8}, -\frac{1}{16}, \ldots \} \]

Subsequential limits: just 0.
\[ \text{Lim sup: } 0 \quad \text{Lim inf: } 0 \]
Converges to 0.

\[ \{ v_n \} = \{ (-1)^n + \frac{1}{n} \} = \{ 0, \frac{3}{2}, -\frac{2}{3}, \frac{5}{4}, -\frac{4}{5}, \frac{7}{6}, \ldots \} \]

Subsequential limits: \{ -1, 1 \}
\[ \text{Lim sup: } 1 \quad \text{Lim inf: } -1 \]
Neither converges nor diverges to either \(+\infty\) or \(-\infty\).