Large deviations for analytic functions with diffusing coefficients

Abstract. We consider a random analytic function $f$ whose zero set is translationally invariant in the plane. This zero set is amazingly “lattice-like”. A recent work of Sodin and Tsirelson shows that it can be matched to a perturbed lattice with Gaussian tails. Moreover, the “hole probability” that a disk of radius $R$ contains no zeros of $f$ decays exponentially in the square of the area of the disk. This asymptotic behavior is also observed in the perturbed lattice model in which lattice points are perturbed by independent complex normal random variables. Allowing the coefficients of $f$ to evolve as independent Ornstein-Uhlenbeck processes, the zero set evolves as a time homogeneous Markov process in the plane. We show that the probability of observing a hole of radius $R$ for time $T$ in the random zero model is exponentially worse than the corresponding probability for the perturbed lattice model in which lattice points evolve as independent Ornstein-Uhlenbeck processes.