1. Let \( C \) be the circle \(|z| = 2\), traversed counterclockwise. Evaluate the integrals

\[
I = \int_C \frac{e^{3/z}}{1 + z} \, dz;
\]
\[
I = \int_C \frac{z + i}{(z - i)^2} \, dz;
\]
\[
I = \int_C \sin \left( \frac{1}{z - 3} \right) \, dz.
\]

Clarify which theorem(s) you use and why they apply.

2. Calculate the value of the following real integral using the methods of complex analysis:

\[
I = \int_0^{2\pi} \frac{\, d\theta}{5 \cos \theta - 3}.
\]

3. Specify the maximal domain of analyticity of the functions:

\[
f(z) = e^{-1/z};
\]
\[
f(z) = \frac{1}{|z|^2 - 5}.
\]

4. Find the first three terms in the Taylor series expansion about \( z_0 = 0 \) of the function

\[
f(z) = \frac{\sin z + 1}{1 + z^2}.
\]

What is the radius of convergence of the above series?

5. Derive two series expansion (Taylor / Laurent) about \( z_0 = 0 \) of the function

\[
f(z) = \frac{z^4}{4z + z^{-1}}.
\]

What is the domain of convergence of each of the series?

6. For what value of \( \lambda \) is there an analytic function whose real part is

\[
u(x, y) = x^3 + \lambda xy^2 + 5y - 10?
\]

Find the corresponding harmonic conjugate function \( v(x, y) \).