HOMEWORK ASSIGNMENT 4

(1) Solve problems 4.3, 4.6, 4.7, 4.8, 4.9, 4.18, 4.19

(2) BONUS question: Show that applying the Gram-Schmidt procedure to the functions $p_0(x) = 1$, $p_1(x) = x$, $p_2(x) = x^2$, ... (as elements in $L^2(-1,1)$) one gets the sequence of Legendre polynomials

$$P_k(x) = \frac{(2k + 1)^{1/2}}{2^{1/2}k!} \frac{d^k}{dx^k}(x^2 - 1)^k$$

(note that the normalization we use is different than that of the book). Hint: This is essentially the same as saying that $P_0$, $P_1$, ... is an orthonormal sequence (explain why). First show that they are orthogonal (if you find the trick, which involves integration by parts, this is not difficult). Finally, you need to evaluate the norm of $P_k(x)$. Again, integration by parts is needed. The following formula (which you do not have to prove) might be useful:

$$\int_{-1}^{1} (x^2 - 1)^k dx = \frac{2^{2k+1}(k!)^2}{(2k + 1)!}.$$