This homework is due Thursday November 14th at the start of class. Remember to write clearly, and justify your solutions. Please make sure to put your name on the first page and to number and staple your pages.

**Problems From The Book:** Axler Chapter 8 problems 3, 25, 28 (hint: for what $p$ of degree at most $n$ is $p(T)e_1 = 0$?), Chapter 9 problems 8, 9 In addition do the following problems:

**Question 1.** (30 points)

(a) Give an operator $T \in \mathcal{L}(\mathbb{C}^4)$ with minimal polynomial $(z - 1)^2(z - 2)$.

(b) What are possible Jordan forms for such a $T$?

(c) What are the possible characteristic polynomials of such a $T$?

**Question 2.** (10 points) Let $T \in \mathcal{L}(\mathbb{C}^5)$ be given by multiplication by the matrix

$$
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 3
\end{bmatrix}.
$$

What is the monic polynomial of smallest degree $p$ so that $p(T)(1,0,0,1,0) = 0$?

**Question 3.** (10 points) Let $T \in \mathcal{L}(\mathbb{R}^2)$ be given by the matrix

$$
\begin{bmatrix}
1 & -1 \\
1 & 1
\end{bmatrix}.
$$

Let $S \in \mathcal{L}(\mathbb{C}^2)$ be given by the same matrix. What are the invariant subspaces of $T$ and $S$?

**Question 4.** Approximately how much time did you spend working on this homework?