This homework is due Thursday October 10th at the start of class. Remember to write clearly, and justify your solutions. Please make sure to put your name on the first page and to number and staple your pages.

**Problems From the Book:** Axler Chapter 2, problems 5 (hint: use the result from 7), 7, 11, 14, and 15.

**Question 1.** Let $V = \{ p \in P(\mathbb{R}) : \deg(p) \leq 2, p(1) = 0 \}$ be a vector space over $\mathbb{R}$. Find a basis of $V$.

**Question 2.** Let $C = \{(1, x, x^2) : x \in \mathbb{R}\}$ be a subset of $\mathbb{R}^3$. What is $\text{span}(C)$?

**Question 3.** Show that a set of non-zero vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ is linearly independent if and only if $\text{span}(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n) = \text{span}(\vec{v}_1) \oplus \text{span}(\vec{v}_2) \oplus \cdots \oplus \text{span}(\vec{v}_n)$.

**Question 4.** For a vector space $V$ and $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in V$ show that

(a) $\text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = \text{span}(\vec{v}_1 + \vec{v}_2, \vec{v}_2 + \vec{v}_3, \vec{v}_3 + \vec{v}_1)$

(b) $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent if and only if $\vec{v}_1 + \vec{v}_2, \vec{v}_2 + \vec{v}_3, \vec{v}_3 + \vec{v}_1$ are.

**Question 5.** Let $U$ and $V$ be vector spaces over a field $\mathbb{F}$. Define their external direct sum by

$U \oplus V = \{(\vec{u}, \vec{v}) : \vec{u} \in U, \vec{v} \in V\}$.

Define addition and scalar multiplication on this set by

$(\vec{u}, \vec{v}) + (\vec{u}', \vec{v}') = (\vec{u} + \vec{u}', \vec{v} + \vec{v}')$

and

$a(\vec{u}, \vec{v}) = (a\vec{u}, a\vec{v})$.

Show that:

(a) $U \oplus V$ is a vector space with these operations.

(b) If $U$ and $V$ are finite dimensional, then so is $U \oplus V$ and $\dim(U) + \dim(V) = \dim(U \oplus V)$.

**Question 6.** Approximately how much time did you spend working on this homework?