Math53: Ordinary Differential Equations  
Autumn 2004  

Homework Assignment 2

Problem Set 2 is due by 2:15p.m. on Monday, 10/11, in MuddChem 101

Problem Set 2:

2.6: 10,14,26,36; 2.9: 20,26,28; 4.3: 4,10,14,26; 4.4: 17 (1st part only); Problem B (see next page)

Note: While the statement of Problem B looks long, most of it is actually a review.

Daily Assignments:

Please review complex numbers, pp181-184, before Thursday, 10/7

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General hint: Doing computations with complex exponentials is usually easier than with real trigonometric functions.
Problem B

Let $p$ and $q$ be two constants. Suppose $\lambda_1$ and $\lambda_2$ are the two roots of the characteristic polynomial

$$\lambda^2 + p\lambda + q = 0$$

associated to the linear homogeneous second-order ODE

$$y'' + py' + qy = 0.$$ 

As stated in class,

$$(e^{(\lambda_1 - \lambda_2)t}(e^{-\lambda_1 t}y))' = e^{-\lambda_2 t}(y'' + py' + qy).$$

Thus, every second-order linear ODE with constant coefficients,

$$y'' + py' + qy = f(t)$$

can be solved in four steps:

*Step 1:* find the roots of the associated characteristic polynomial (1);
*Step 2:* multiply both sides of (3) by $e^{-\lambda_2 t}$;
*Step 3:* use (2) to compress LHS of the resulting expression and to obtain

$$\left(e^{(\lambda_1 - \lambda_2)t}(e^{-\lambda_1 t}y)\right)' = e^{-\lambda_2 t}f(t);$$

*Step 4:* solve (4) for $y$ by integrating twice.

This approach mimics the integrating factor method for solving linear first-order ODEs, though it works only for constant $p$ and $q$. Its advantage over the methods described in Sections 4.3 and 4.5 of the text is that

1. it works the same way whether or not $\lambda_1$ and $\lambda_2$ are distinct;
2. it works the same way no matter what $f$ looks like.

Use the above second-order integrating factor method to find the real (not complex) general solutions of

(a) $y'' + 4y = 4\cos 2t$;
(b) $y'' + 5y' + 4y = t \cdot e^{-t}$. 