Math120 – Writing in the major
Grading report
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The first draft was graded out of 10 points. The division was 6 points for mathematical correctness and 4 points for writing. However, I will very likely place more weight on writing on future drafts. After grading the first drafts, I found weaknesses that appeared over and over again. I enumerate them below.¹

IMPORTANT

1. Consider this handout to be part of your feedback about draft 1. In particular, you should address the issues enumerated here in subsequent drafts, even if I didn’t specifically mark them in your paper.

2. When handing in future drafts, also hand in your graded version of the previous draft. I will return both.

Organization and clarity

• Your paper should stand on its own. The reader shouldn’t need to know that you wrote it to fulfill an assignment in order to understand it. In particular, if you choose to follow the suggested outline, the reader shouldn’t have to use the assignment as a crib sheet to follow your paper.

• Your paper should have a descriptive title. This should not be a generic title like “my math120 paper,” but it should rather tell the reader what your paper is about.

• The reader should always know what you’re up to. You shouldn’t plunge into heavy argument without saying what your goal is. For example, write “now we prove that this map is injective,” and then prove that it is injective. When you’re done proving that it is injective, say something like “Hence, the map is injective.” In other words (and oversimplifying): “Say what you’re gonna do, do it, then say what you did.”

• It does not hurt to remind the reader of important definitions. One way to do this is to have a separate “preliminaries” section.

• If the notation you are using is not standard, you have to define it. In particular:
  - The notation for the congruence class of $x$ modulo $n$ is usually written $\overline{x}$. However, this notation does not mention $n$, so it lends itself to confusion when there are different moduli floating around. You can use less ambiguous notation, such as $\overline{x}_n$ or $[x]_n$, but if you do, make sure to define it.

¹I don’t necessarily practice what I preach in this handout. Lo siento.
– The use of “∗” for multiplication and “∧” for exponentiation is not standard in mathematics (even if it is common in programming languages). Avoid it, or define it if you must use it.

– In the textbook, the symbol mod does not stand for an operation, even if it does in the standard notation of other subjects, like computer science. So, we do not write \(a \mod m = b \mod m\). Instead we have a relation of congruence modulo \(m\), and this is written \(a \equiv b \pmod{m}\). As before, you can use mod as an operation, but if you do that you must define it explicitly as one.

• Avoid adding in hypotheses that you don’t ever use, even if you can assume them “without loss of generality.” If on rereading you find that you never used an assumption, then remove it. This will make your paper more readable.

• In product notation, include the limits. That is, write something like \(\prod_{i=1}^{k} i\) instead of \(\prod_{i}\).

• Be clear about exactly where a proof ends. You can indicate this, in decreasing order of clarity, with a symbol (like ■), with extra spacing, with “QED,” or with a sentence (“This finishes the proof.”). This makes it easier for the reader to skip the proof and follow the logic of the paper on a first reading.

• Avoid perverse notation. For instance, you wouldn’t use the letter \(\varphi\) to define a new thing when it already stands for, say, the Euler phi function, would you? Hah! Well, you get the idea.

• If you say “hence,” “therefore,” or “thus,” it means that the justification comes before. If that’s not the case, you shouldn’t be using this word and should instead use something like “we now show that...”

• Consider labeling your intermediate results and equations for easy reference. That way you don’t have to keep writing “by a previous result” or “as we saw earlier,” which is ambiguous. Instead, you write “by Lemma 3” or “by equation (4.18),” which is clear.

Math

• The notation \(\mathbb{Z}/m\mathbb{Z}\) stands for the group whose elements are congruence classes modulo \(m\) and whose operation is addition. In particular, \(\mathbb{Z}/m\mathbb{Z}\) is not equal as a set to \(\{0, 1, 2, \ldots, m - 1\}\), even though this is a complete set of representatives of the classes contained in \(\mathbb{Z}/m\mathbb{Z}\).

• The notation \((\mathbb{Z}/m\mathbb{Z})^\times\) stands for the subset of \(\mathbb{Z}/m\mathbb{Z}\) consisting of elements which have a multiplicative inverse. This set is a group under multiplication. Notice in particular that since the operations are different, it is not accurate to say that \((\mathbb{Z}/m\mathbb{Z})^\times\) is a subgroup of \(\mathbb{Z}/m\mathbb{Z}\), even if sometimes we say that when we’re sloppy.

• There is an important difference between these two statements about groups \(A\) and \(B\):

1. \(A\) is isomorphic to \(B\).
2. The map \(f : A \rightarrow B\) (say, a map constructed earlier) is an isomorphism.

The second statement is stronger, since it tells you that not only does there exist an isomorphism between the groups, but that the isomorphism is given by a particular map. This can be useful (or indispensable) knowledge about the map \(f\).
• On relative primality. The statement that some integers \( a_1, \ldots, a_k \) are relatively prime means that the greatest common divisor of them all is 1. This is strictly weaker than saying that the numbers \( a_1, \ldots, a_k \) are pairwise relatively prime; the latter statement means that for any \( 1 \leq i < j \leq k \) we have that \( a_i \) and \( a_j \) are relatively prime. Make sure to be clear about which of the two you mean in a given context.

• If you have a map \( f \) which is a homomorphism, the cleanest way to prove that it is injective is to show that its kernel is trivial. By this I don’t mean that it will be less work than showing it by the usual definition (if \( f(x) = f(y) \) then \( x = y \)) but that it will usually be more readable.

• In a proof by induction, you should be clear about what quantity you are inducting on.

• On restricting a map. If \( f \) is a map from \( S \) to \( T \) and \( S' \) is a subset of \( S \), one can define the restriction of \( f \) to \( S' \). So, it is valid to say “let \( f' \) be the restriction of \( f \) to \( S' \).” However, if we additionally have a subset \( T' \) of \( T \), one can’t simply say “restrict \( f \) to \( f' : S' \to T' \).” This is because after restricting the domain of \( f \) to \( S' \) we must verify that this restriction admits the codomain \( T' \), that is, we must check that \( f'(S') \subseteq T' \). Only after doing that can we say that \( f' \) is a map from \( S' \) to \( T' \).

Typesetting and style

• You are strongly encouraged to turn in double-spaced papers; that way it is easier to insert comments.

• Avoid the use of symbols (such as \( \exists, \forall, \therefore, \iff \) ) in the main text of your paper. Instead, use English. In particular, the verb of a sentence should never be a symbol. So, don’t write: “Therefore, the value of the function \( > 2 \).” Do not even write “Therefore, the value of the function is \( > 2 \).” Instead, write “Therefore, the value of the function is greater than 2.” However, it is fine for a whole subordinate clause to be stated in mathematical notation, as long as you don’t go overboard. For instance, one can write “Therefore, we have that \( f(x) > 2 \).”

• Avoid starting a sentence with mathematical symbols. So, instead of writing “\( f(x) = 2 \), and the result follows.” write (for instance) “We have that \( f(x) = 2 \), and the result follows.”

• Avoid the use of words like “evident” or “trivial” and other similarly arrogant adjectives. It is usually OK to write that something is “clear,” though your reader might disagree.

• The word “theorem” is only capitalized when you are referring to a specific theorem. For instance, you should write “I will prove a nice theorem,” but you should write “I know it’s so thanks to Theorem 26.18.” Similar remarks go for “lemma,” “definition,” “corollary,” etc.

• Choose to write either as “I” or as “we” and stick to it consistently. So, if you began your paper with “We will show that....” you shouldn’t write elsewhere something like “Now, I construct a function....”

• Do not write in the imperative. Your assignment was written in the imperative because it is telling you what to do. You are not supposed to tell your reader what to do. (There are exceptions to this; consider “consider.”)