Math 51h Homework 6 (due Friday, November 9)

By Monday, you should have read all of chapter 8. You should read theorem 8.12 and its proof in §8.23. Next week, you should also read §9.6 until the middle of page 297, §9.9, and §9.14.

§8.14, p 262: 7abd
§8.16, p 268: 8, 9
§8.22, Page 275: 14
§8.24, Page 281: 1, 5
§9.8, Page 303: 6, 9, 10
§9.13, Page 313: Find (but do not classify) the critical points in 1,4,7,10,13. Also:

1. Let $S$ and $T$ be convex sets in $\mathbb{R}^N$. Prove that the intersection $S \cap T$ of $S$ and $T$ is also convex.

2. A certain function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ has $xyz = 6$ as one of its level sets. Also,
   \[
   \frac{\partial f}{\partial x}(1,2,3) = 12
   \]
   Find $\nabla f(1,2,3)$.

3. The temperature at point $(x,y)$ on a table is $f(x,y) = x^3 + xy + y^3$. An ant crawls along the table in such a way that its temperature is always 1. At time $t = 0$, the ant is at the point $(0,1)$, its $x$-coordinate is increasing, and its speed is 7. Find the ant’s velocity at time $t = 0$.

4. Let $f$ and $g$ be differentiable functions from $\mathbb{R}^n$ to $\mathbb{R}^n$ such that
   a. $f(0) = g(0) = 0$.
   b. $f(x) = y$ if and only if $g(y) = x$.
   Prove that $Df(0) \circ Dg(0) = I$ where $I$ is the $n \times n$ identity matrix (i. e., the $n \times n$ matrix $I$ with $I_{ii} = 1$ and $I_{ij} = 0$ for $i \neq j$).

5. A linear combination $\sum_{i=1}^{k} a_i x_i$ of points $x_1, \ldots, x_k$ is called a convex combination of the $x_i$ if
   (1) each coefficient $a_i$ is $\geq 0$, and
   (2) $\sum_{i=1}^{k} a_i = 1$.
   Let $S$ be a convex set in $\mathbb{R}^N$. Prove that if $x_1, \ldots, x_k$ are points in $S$, then any convex combination $p = \sum a_i x_i$ of those points is also in $S$.
   (It may help you to consider first the cases $k = 2$ and $k = 3$.)