1. If $Q$ is the cube $\{x : -1 \leq x_1, x_2 \leq 1\}$, then denote by $\chi_Q(x)$ the function which equals 1 when $x \in Q$, 0 when $x \notin Q$. Compute the Radon transform of $f(x) = x_1x_2\chi_Q(x)$.

2. Suppose that $\{p_1, \ldots, p_n\}$ is a collection of $n$ distinct points on the unit circle. For $i \neq j$, the line segment $L_{ij}$ connecting $p_i$ to $p_j$ is part of an entire line $\ell_{ij}$, to which we assign the parameters $t_{ij}, \omega_{ij}$ in the usual way. Fix real numbers $r_{ij} = r_{ji}, 1 \leq i \neq j \leq n$. Show that there exists a function $f$ which is nonzero precisely on the line segments $L_{ij}$ and such that $\mathcal{R}f(t_{ij}, \omega_{ij}) = r_{ij}$.

3. Carefully supply the steps in the justification of the following facts. In some of the following it may be helpful to use the fact that the space $C_0^\infty(\mathbb{R})$, consisting of all smooth infinitely differentiable functions which vanish outside a compact set, is dense in both $L^1(\mathbb{R})$ and also in $L^2(\mathbb{R})$; you may take this as known.

   a) If $f \in L^1(\mathbb{R})$, then $\hat{f}(\xi)$ is a continuous function. (Use the inequality $||\hat{f}||_{C^0} \leq ||f||_{L^1}$.)

   b) If $f_j \rightarrow f$ in $L^2(\mathbb{R})$, then $||f_j||_{L^2} \rightarrow ||f||_{L^2}$.

   c) Fill in all the details of the Plancherel theorem, that $||f||_{L^2}^2 = (2\pi)^{-1}||\hat{f}||_{L^2}^2$ makes sense and holds for every $f \in L^2$, starting with the fact that this equality holds (by the easy calculation done in class) for $f \in C_0^\infty$.

4. Suppose that $m(\xi)$ is a bounded function on the line (not necessarily continuous). Define the mapping

\[(A_m f)(x) = \frac{1}{2\pi} \int e^{ix\xi}m(\xi)\hat{f}(\xi) \, d\xi.\]

(In other words, $A_m = \mathcal{F}^{-1} \circ m \circ \mathcal{F}$, where the $m$ in the center here means multiplication by the function $m$.) Prove that $A_m : L^2 \rightarrow L^2$ is a continuous mapping, or in other words, that for every $f \in L^2$, $A_m f \in L^2$ and $||A_m f||_{L^2} \leq C||f||_{L^2}$ for some constant $C$ independent of $f$. 

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