FAKE TEST

- Complete the following fake problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes fake exam. No calculators or other electronic aids will be permitted.
- In order to receive full fake credit, please show all of your work and justify your fake answers. You do not need to simplify your fake answers unless specifically fake instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this fake exam. Do not unstaple or detach pages from this fake exam.
- Please sign the following:
  “On my honor, I have neither given nor received any aid on this fake examination. I have furthermore abided by all other aspects of the honor code with respect to this fake examination.”

Signature: ______________________________

The following boxes are strictly for fake grading purposes. Please do not mark.

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(1) (10 points) Answer the following clearly, completely, and concisely.

- Define ‘dimension.’

- List the qualities a matrix must satisfy to be in reduced row echelon form.
(2) (15 points) Determine whether each statement is true or false; unless otherwise stated, any function below is arbitrary. If the statement is true, cite your reasoning. If it is false, provide a counterexample.

(a) If $A$ is an $n \times n$ matrix whose columns are linearly independent, then $C(A) = \mathbb{R}^n$.

(b) If $A$ and $B$ are two $3 \times 3$ matrices so that $rref(A)$ and $rref(B)$ both have 2 pivot columns, then $A$ is row equivalent to $B$.

(c) If $A$ is an $r \times c$ matrix so that $rref(A)$ has a row of zeros, then for any $b \in \mathbb{R}^r$ the system $A \overline{x} = \overline{b}$ is inconsistent.
(3) (15 points) Let $A$ be the matrix

\[
\begin{pmatrix}
1 & 1 & -1 \\
2 & 3 & 2 \\
2 & 1 & -6 \\
3 & 5 & 5 \\
\end{pmatrix}
\]

Find bases for $N(A)$ and $C(A)$. 
(4) (10 points) Brenda has a system of equations she calls $S$, and she calls the coefficient matrix of her system $C$. Brenda has (correctly) computed

$$\text{rref}(C) = \begin{pmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$ 

Brenda tells you that she has found a solution $\vec{x}_p$ to her system. Give an explicit expression for a general solution to Brenda’s system (your answer will involve $\vec{x}_p$).
(5) (10 points) George has a system of equations he calls $R$, and he calls the coefficient matrix of his system $B$. George has (correctly) computed

\[ \text{rref}(B) = \text{rref}(C), \]

where $C$ is the matrix in the previous problem. George claims that $R$ does not have infinitely many solutions. Can both Brenda and George be telling the truth? If not, explain why. If so, give examples of augmented matrices which could correspond to the systems $S$ and $R$, with $S$ having at least one solution and $R$ not having infinitely many solutions.
(6) (15 points) Alton has a collection of vectors \( \{\vec{v}_1, \cdots, \vec{v}_s\} \) from a subspace \( W \) which he claims is a basis for \( W \). Rachel has a different collection of vectors \( \{\vec{w}_1, \cdots, \vec{w}_t\} \) from \( W \) which she claims is a basis for \( W \).

- Suppose both Alton and Rachel are telling the truth. What can you say about \( s \) and \( t \)?

- Suppose that Alton is telling the truth and that \( s < t \). What condition for being a basis will Rachel’s collection necessarily fail? Explain your answer.
(7) (10 points) Suppose that \( T \) is a linear transformation from \( \mathbb{R}^c \) to \( \mathbb{R}^r \). Show that if \( \{ \vec{v}_1, \cdots, \vec{v}_s \} \) is a linearly dependent collection in \( \mathbb{R}^c \) then \( \{ T(\vec{v}_1), \cdots, T(\vec{v}_c) \} \) is a linearly dependent collection in \( \mathbb{R}^r \).
(8) (15 points) Suppose that \( \overrightarrow{v} \) is a vector in \( \mathbb{R}^3 \). Show that the collection of all vectors in \( \mathbb{R}^3 \) orthogonal to \( \overrightarrow{v} \) is a subspace of \( \mathbb{R}^3 \). That is, show that the collection

\[ W = \{ \overrightarrow{x} \in \mathbb{R}^3 : \overrightarrow{x} \cdot \overrightarrow{v} = 0 \} \]

is a subspace of \( \mathbb{R}^3 \).