Notes for sections 2007. 3. 15

§ 7.6 P549

1\. (a) $\frac{dx}{dt} = -0.05x + 0.0001xy$

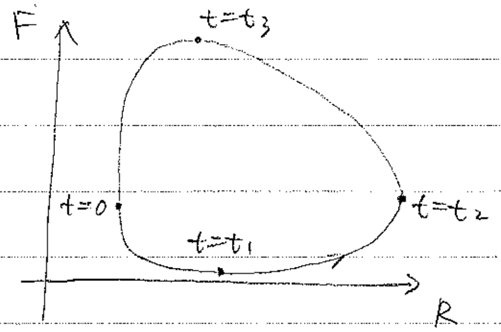
If $y=0$, we have $\frac{dx}{dt} = -0.05x$, which indicates that in the absence of y, x declines at a rate proportional to itself. So x represents the predator population and y represents the prey population.

The growth of the prey population, $0.1y$ (from $\frac{dy}{dt} = 0.1y - 0.005xy$), is restricted only by encounters with predators (the term $-0.005xy$). The predator population increases only through the term $0.0001xy$; that is, by encounters with the prey and not through additional food sources.

(b) $\frac{dy}{dt} = -0.015y + 0.00008xy$

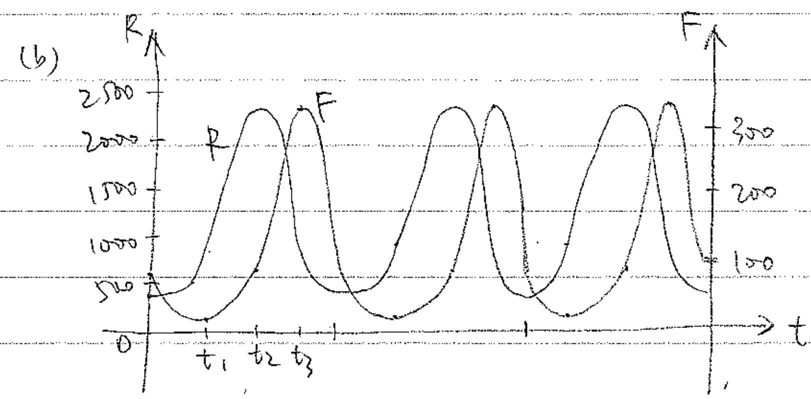
If $x=0$, we have $\frac{dy}{dt} = -0.015y$, which indicates that in the absence of x, y would decline at a rate proportional to itself. So y represents the predator population and x represents the prey population. The growth of the prey population, $0.2x$ (from $\frac{dx}{dt} = 0.2x - 0.0002x^2 - 0.006xy$ $= 0.2x(1-0.001x) - 0.006xy$, is restricted by a carrying capacity of 1000 (from the term $1-0.001x = 1 - \frac{1}{1000}x$) and by encounters with predators (the term $-0.006xy$). The predator population increases only through the term $0.00008xy$; that is, by encounters with the prey and not through additional food sources.

3. (a) At $t=0$, there are about 300 rabbits and 100 foxes.



At $t=t_1$, the number of foxes reaches a minimum of about 20 while the number of rabbits is about 1000.
At $t=t_2$, the number of rabbits reaches a maximum of about 2400, while the number of foxes rebounds to 100.
At $t=t_3$, the number of rabbits decreases to about 1000 and the number of foxes reaches a maximum of about 315.
As t increases, the number of foxes decreases greatly to 100, and the number of rabbits decreases to 300 (The initial populations), and the cycle starts again.

(b)



9 (a) Letting $W=0$ gives us $\frac{dR}{dt} = 0.08R(1-0.0002R)$

$\frac{dR}{dt} = 0 \iff R=0$ or 5000

Since $\frac{dR}{dt} > 0$ for $0 < R < 5000$, we would expect the rabbit population to increase to 5000 for these values of R.

Since $\frac{dR}{dt} < 0$ for $R > 5000$, we would expect the rabbit population to decrease to 5000 for these values of R.

Hence, in the absence of wolves, we would expect the rabbit population to stabilize at 5000.

(b) R and W are constants $\Rightarrow R' = 0$ and $W' = 0$

$$\Rightarrow \begin{cases} 0 = 0.08R(1-0.0002R) - 0.001RW \\ 0 = -0.02W + 0.00002RW \end{cases}$$

$$\Rightarrow \begin{cases} 0 = R[0.08(1-0.0002R) - 0.001W] \\ 0 = W(-0.02 + 0.00002R) \end{cases}$$

The second equation is true if $W=0$ or $R = \frac{0.02}{0.00002} = 1000$.

If $W=0$ in the first equation, then either $R=0$ or $R = \frac{1}{0.0002} = 5000$

If $R=1000$, then $0 = 1000[0.08(1-0.0002\cdot 1000) - 0.001W]$

$\iff 0 = 80(1-0.2) - W \iff W = 64$

So Case (i) $W=0, R=0$, both populations are zero

Case (ii) $W=0, R=5000$ see part (a)

Case (iii) $W=64, R=1000$, the predator/prey interaction balances and the populations are stable.

(c) The populations of wolves and rabbits fluctuate around 64 and 1000, respectively, and eventually stabilize at those values.

(d)