Notes to sections 2007. 2. 6

§ 7.1 P503.

7. (a) Since the derivative $y' = -y^2$ is always negative (or 0 if $y=0$), the function $y$ must be decreasing (or equal to 0) on any interval on which it is defined.

(b) $y = \frac{1}{x+c} \Rightarrow LHS = y' = -\frac{1}{(x+c)^2}$

$RHS = -y^2 = -\frac{1}{(x+c)^2}$

so $LHS = RHS$

(c) $y=0$ is a solution of $y' = -y^2$ that is not a member of the family in part (b).

(d) If $y(x) = \frac{1}{x+c}$, then $y(0) = \frac{1}{0+c} = \frac{1}{c}$

Since $y(0) = 0.5$, $\frac{1}{c} = 0.5 \Rightarrow c = 2$, so $y = \frac{1}{x+2}$

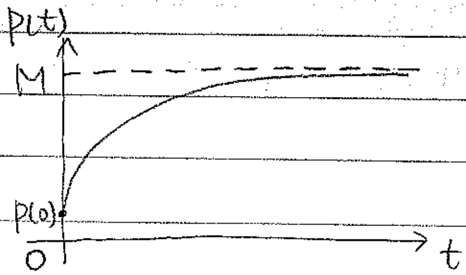
11 (a) This function is increasing and also decreasing. But $\frac{dy}{dt} = e^t(y-1)^2 \geq 0$ for all $t$, implying that the graph of the solution of the differential equation cannot be decreasing on any interval.

(b) When $y=1$, $\frac{dy}{dt} = 0$, but the graph doesn't have a horizontal tangent line.

13. (a) P increases most rapidly at the beginning, since there are usually many simple, easily-learned subskills associated with learning a skill. As $t$ increases, we would expect $\frac{dP}{dt}$ to remain positive, but decrease. This is because as time progresses, the only points left to learn are the more difficult ones.

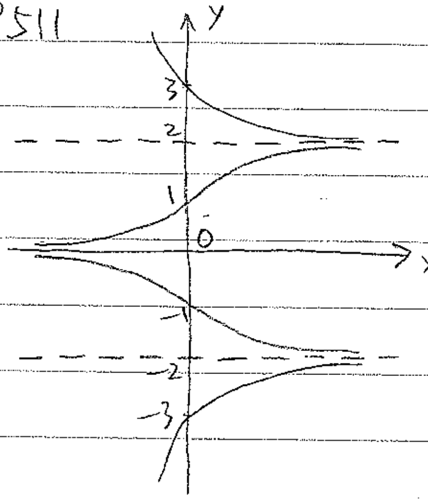
(b) $\frac{dP}{dt} = k(M-P)$ is always positive, so the level of performance P is increasing. As P gets close to M, $\frac{dP}{dt}$ gets close to 0; that is, the performance levels off, as explained in part (a).

(c)




§ 7.2 P511

1. (a)




(b) It appears that the constant function $y=0$, $y=-2$ and $y=2$ are equilibrium solutions. Note that these three values of $y$ satisfy the given differential equation $y' = y\cdot(1-\frac{1}{4}y^2)$

3. $y' = y-2$ The slopes at each point are independent of $x$, so the slopes are the same along each line parallel to the x-axis. Thus, III is the direction field for this equation. Note that for $y=2$, $y'=0$

4. $y' = x(2-y)$. The slope equals 0 on the lines $x=0$ and $y=2$. Direction field I satisfies these conditions.

5. $y' = x+y-1$ The slope equals 0 on the line $y=-x+1$. Direction field IV satisfies this condition. Notice also that on the line $y=-x$ we have $y'=-1$, which is true in IV

6. $y' = \sin x \sin y$ The slope equals 0 on the lines $x=0$ and $y=0$, and $y'>0$ for $0<x<\pi$, $0<y<\pi$, Direction field II satisfies these conditions.

19. (a) $y' = F(x,y) = y$ and $y(0) = 1 \Rightarrow x_0 = 0 \quad y_0 = 1$

(i) $h = 0.4 \Rightarrow x_1 = x_0 + h = 0 + 0.4 = 0.4$

$y_1 = y_0 + hF(x_0, y_0) = 1 + 0.4 \times 1 = 1.4$

so $y(0.4) \approx y_1 = 1.4$

(ii) $h = 0.2 \Rightarrow x_1 = x_0 + h = 0.2, \quad x_2 = x_1 + h = 0.4$

$y_1 = y_0 + hF(x_0, y_0) = 1 + 0.2 \times 1 = 1.2$

$y_2 = y_1 + hF(x_1, y_1) = 1.2 + 0.2 \times 1.2 = 1.44 \approx y(0.4)$

(iii) $h = 0.1 \Rightarrow x_1 = x_0 + h = 0.1$, similarly $x_2 = 0.2, \ x_3 = 0.3, \ x_4 = 0.4$.

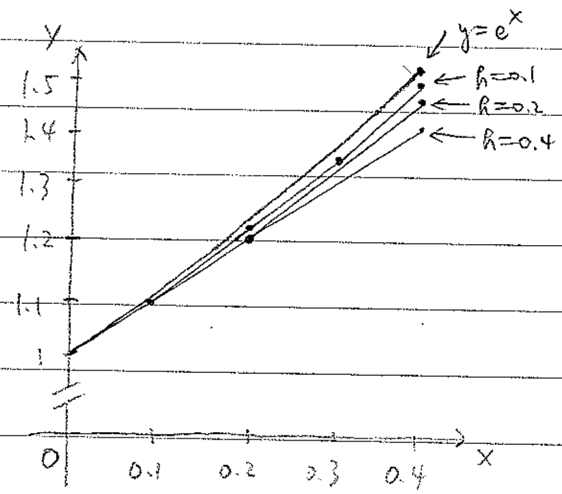
$y_1 = y_0 + hF(x_0, y_0) = 1 + 0.1 \times 1 = 1.1$

$y_2 = y_1 + hF(x_1, y_1) = 1.1 + 0.1 \times 1.1 = 1.21$

$y_3 = y_2 + hF(x_2, y_2) = 1.21 + 0.1 \times 1.21 = 1.331$

$y_4 = y_3 + hF(x_3, y_3) = 1.331 + 0.1 \times 1.331 = 1.4641 \approx y(0.4)$

(b)




We see that the estimates are underestimates since they are all below the graph of $y = e^x$

(c) (i) For $h = 0.4$, error = exact value − approximate value $= e^{0.4} - 1.4 \approx 0.0918$

(ii) For $h = 0.2$, error = exact value − approximate value $= e^{0.4} - 1.44 \approx 0.0518$

(iii) For $h = 0.1$, error = exact value − approximate value $= e^{0.4} - 1.4641 \approx 0.0277$

Each time the step is halved, the error estimate also appears to be halved approximately.

23. $h=0.1 \quad x_0=0 \quad y_0=1 \quad F(x,y)=y+xy$

Note that $x_1=x_0+h=0.1, \quad x_2=0.2, \quad x_3=0.3, \quad x_4=0.4$

$y_1=y_0+hF(x_0,y_0)=1+0.1\,F(0,1)=1+0.1(1+0\times1)=1.1$

$y_2=y_1+hF(x_1,y_1)=1.1+0.1\,F(0.1,1.1)=1.1+0.1(1.1+0.1\times1.1)=1.221$

$y_3=y_2+hF(x_2,y_2)=1.221+0.1\,F(0.2,1.221)=1.221+0.1(1.2+0.2\times1.221)=1.36752$

$y_4=y_3+hF(x_3,y_3)=1.36752+0.1\,F(0.3,1.36752)$

$=1.36752+0.1(1.36752+0.3\times1.36752)=1.5452976$

$y_5=y_4+hF(x_4,y_4)=1.5452976+0.1\,F(0.4,1.5452976)$

$=1.5452976+0.1(1.5452976+0.4\times1.5452976)=1.761639264$

Thus, $y(0.5)\approx y_5=1.7616$