Notes to Sections   2007. 02. 08

A few words about probability :

We don't have too much time to talk about probability in sections. Here are two points I want to stress :

(1) Two types of distributions you have to be familiar with :

Exponential distribution : as in examples 2, 3, 4 in the text

It models waiting times and equipment failure times (and so on). The form of the density function is given on page 488. According to example 3, the mean is $\frac{1}{c}$, where c is the constant appearing in the density function. The mean is important because if you know the distribution is exponential and you know the mean, the whole distribution is completely known.

Normal distribution : as in example 5

It models test scores, heights, weights, and so on. The form of the density function is given on page 491. Unlike the exponential distribution, the distribution is determined by two parameters, namely the mean and the standard deviation.

There are many daily / weekly homework problems for you to get familiar with the two important distributions.

(2) The difference between mean and median.

Intuitively, mean is the "average" value of the random variable. By definition, mean $\mu = \int_{-\infty}^{+\infty} x f(x)\, dx$.

However, median is a number which separates the probability half by half, namely, the probability for the random variable to be less than the median and the probability for the random variable to be larger than the median are both $\frac{1}{2}$.
The mean and median are the same for normal distributions because the density function is symmetric. However, they are indeed different in exponential distributions. If you haven't done so, you should really try this: if the exponential density function is given as in example 3, then the mean is $\frac{1}{c}$ and the median is $\frac{\ln 2}{c}$.

A mini course about geometric series:

- A few examples of geometric series:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\cdots = \sum_{n=1}^{\infty}\frac{1}{2^n}$$

$$2+2+2+\cdots = \sum_{n=1}^{\infty} 2$$

$$1+(-1)+1+(-1)+1+(-1)+\cdots = \sum_{n=1}^{\infty}(-1)^{n+1}$$

$$5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+\cdots$$

$$a+ar+ar^2+ar^3+\cdots = \sum_{n=1}^{\infty} ar^{n-1} \quad (a\neq 0)$$

$$2+4+8+16+32+\cdots = \sum_{n=1}^{\infty} 2^n$$

- Criterion for recognizing a geometric series

CONSTANT COMMON RATIO !

namely: the radios of all pairs of neighboring terms are the same number

or: each term is obtained by multiplying the preceding term by the same number

You should know how to find out the common ratio! (say, r)

- convergent or divergent

Fact: the geometric series (with common radio r) is
$\begin{cases} \text{convergent if } |r|<1 \\ \text{divergent if } |r|\geq 1 \end{cases}$

- sum of convergent geometric series

Formula: sum of convergent geometric series
$$= \frac{\text{first term}}{1-(\text{common ratio})}$$

Note: cannot use this formula for divergent geometric series.

Examples:

① $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \cdots$

ratio $= -\frac{2}{3} \Rightarrow$ convergent sum $= \frac{5}{1-(-\frac{2}{3})} = \frac{5}{\frac{5}{3}} = 3$

② $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n-1}$ ratio $= \frac{2}{3} \Rightarrow$ convergent sum $= \frac{5}{1-\frac{2}{3}} = \frac{5}{\frac{1}{3}} = 15$

③ $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}}$ ratio $= \frac{\pi}{3} \Rightarrow$ divergent

④ $\sum_{n=1}^{\infty} 2^{2n} 3^{1-n}$ rewrite it as $\sum_{n=1}^{\infty} 3 \cdot \left(\frac{2^2}{3}\right)^n = \sum_{n=1}^{\infty} 3 \cdot \left(\frac{4}{3}\right)^n$

ratio $= \frac{4}{3} \Rightarrow$ divergent

⑤ write the number $2.3\overline{17}$ as a fraction

$2.3\overline{17} = 2.3 + \frac{17}{10^3} + \frac{17}{10^5} + \frac{17}{10^7} + \cdots$

ratio $= \frac{1}{10^2} \Rightarrow$ convergent $\Rightarrow 2.3\overline{17} = 2.3 + \frac{17}{10^3} \Big/ \left(1-\frac{1}{10^2}\right) = \frac{23}{10} + \frac{17}{990} = \frac{1147}{495}$

⑥ Find the values of x for which the series converges and find the sum

$\sum_{n=1}^{\infty} \frac{x^n}{3^n}$ ratio $= \frac{x}{3}$ series is convergent $\Leftrightarrow \left|\frac{x}{3}\right| < 1 \Leftrightarrow |x| < 3$

So $-3 < x < 3$. sum $= \frac{x}{3} \Big/ \left(1-\frac{x}{3}\right) = \frac{x}{3-x}$