Section Notes 11/15/07

1. Antiderivatives (Indefinite integrals)

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Not hard if you are familiar with antiderivatives of elementary functions. When no extra conditions are given, the antiderivative can only be determined up to a constant.

31. Note that the antiderivative of a constant function is a linear function whose graph is a straight line. So the graph of f in this problem should be three connected line segment. $f(0) = -1$ gives you the starting position.

51. Let's assume the constant deceleration is k km/h^2, which means $a(t) = -k$. What you know is:

initial velocity $V(0) = 100$

initial position $S(0) = 0$

Antiderivatives:

$$\left.\begin{array}{l} a(t) = -k \Rightarrow V(t) = -kt + C \\ V(0) = 100 \end{array}\right\} \Rightarrow C = 100 \Rightarrow V(t) = -kt + 100$$

$$\left.\begin{array}{l} \Rightarrow S(t) = -\frac{1}{2}kt^2 + 100t + D \\ S(0) = 0 \end{array}\right\} \Rightarrow D = 0 \Rightarrow S(t) = -\frac{1}{2}kt^2 + 100t$$

What we want to compute:

when the car stops, it's no more than 80 m = 0.08 km away from the initial position.

Question 1. When does the car stop?

$$V(t) = -kt + 100 = 0 \Rightarrow t = \frac{100}{k}$$

Question 2. What's the position of the car when it stops?

$$S\left(\frac{100}{k}\right) = -\frac{1}{2}k \cdot \left(\frac{100}{k}\right)^2 + 100 \cdot \left(\frac{100}{k}\right) = \frac{5000}{k}$$

Question 3. How to make it no more than 0.08 km ?

$$\frac{5000}{k} < 0.08 \Rightarrow k > \frac{5000}{0.08} = 62500 \ km/h^2$$

Answer: The deceleration should be larger than $62500 \ km/h^2$

55. (a) Fix units: $90 \ mi/h = 90 \times \frac{5280}{3600} \ ft/s = 132 \ ft/s$

Analyze the behavior of the car:

First part: increase speed from $0 \ ft/s$ to $132 \ ft/s$ at the rate of $4 \ ft/s^2$

Second part: run at the speed of $132 \ ft/s$ for $15 \ min = 15 \times 60 = 900 \ sec$.

Compute the distance for the acceleration part:

$$\left.\begin{array}{l} a(t) = 4 \Rightarrow v(t) = 4t + C \\ V(0) = 0 \end{array}\right\} \Rightarrow C = 0 \Rightarrow V(t) = 4t$$

$$\left.\begin{array}{l} \Rightarrow s(t) = 2t^2 + D \\ S = 0 \end{array}\right\} \Rightarrow D = 0 \Rightarrow S(t) = 2t^2$$

time for the acceleration part: $v(t) = 132 \Rightarrow 4t = 132 \Rightarrow t = 33 \ sec$

distance for the acceleration part: $S(33) = 2 \cdot 33^2 = 2178 \ ft$

Compute the distance for the cruise part:

$$132 (ft/s) \times 900 (s) = 118800 \ ft$$

Total distance: $2178 + 118800 = 120978 \ ft$

(b) Hint:

1st stage: acceleration: the same as above.

33 seconds, 2178 ft.

2nd stage: Cruise, constant speed at $132 \ ft/s$.

time unknown (temporarily)

3nd stage: deceleration: the opposite procedure to the acceleration stage. 33 seconds, 2178 ft.

total time: $15 \min = 15 \times 60 \sec = 900 \sec$

time during the 2nd stage: $900 - 33 - 33 = 834 \sec$.

distance at the 2nd stage: $132 \times 834 = 110088$ ft

Total distance: $2178 + 110088 + 2178 = 114444$ ft

[2] Estimate areas under a curve (estimate definite integrals)

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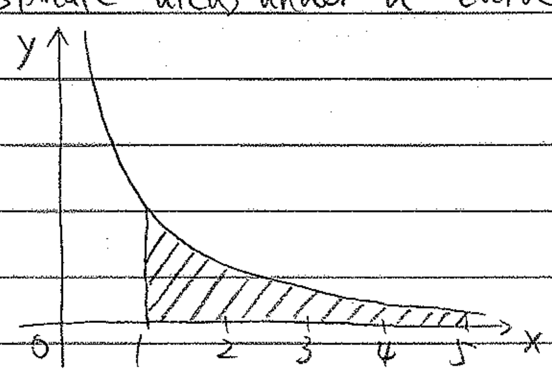


function: $f(x) = \frac{1}{x}$ (graph on left)

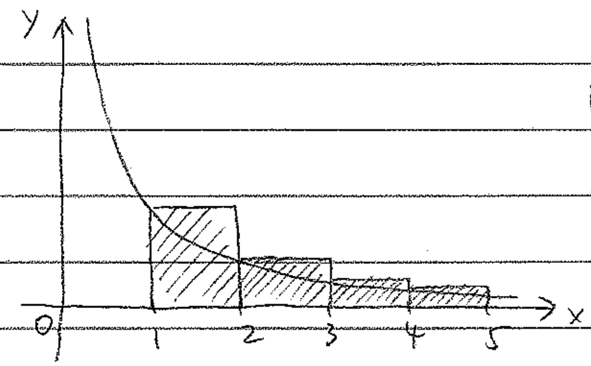
number of intervals: $n = 4$

interval: $[1, 5]$

Compute: size of each piece: $\Delta x = 1$

separating points:

$x_0 = 1,\ x_1 = 2,\ x_2 = 3,\ x_3 = 4,\ x_4 = 5$



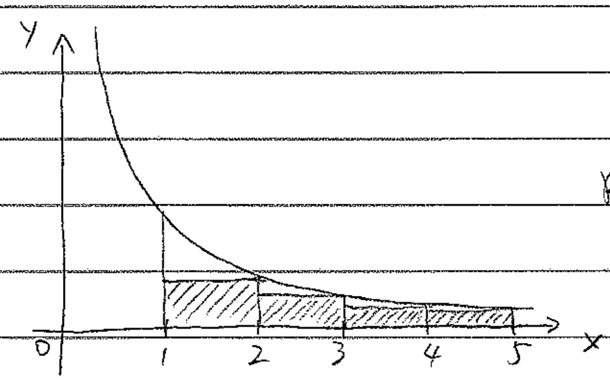
Using left endpoints:

$L_4 = f(1) \cdot \Delta x + f(2) \cdot \Delta x + f(3) \cdot \Delta x + f(4) \cdot \Delta x$

$= (f(1) + f(2) + f(3) + f(4)) \cdot \Delta x$

$= (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}) \cdot 1 = \frac{25}{12}$

overestimate



Using right endpoints

$R_4 = f(2) \cdot \Delta x + f(3) \cdot \Delta x + f(4) \cdot \Delta x + f(5) \cdot \Delta x$

$= (f(2) + f(3) + f(4) + f(5)) \cdot \Delta x$

$= (\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}) \cdot 1 = \frac{77}{60}$

underestimate

General rule: increasing function: L_n is underest. R_n is overest.

decreasing function: L_n is overest R_n is underest.

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It's OK even if the function is not given. All we need to estimate the integral is the values of the function at the separating points (endpoints of the small intervals).

Interval : $[0, 25]$ number of small intervals: $n=5$

Size of small intervals : $\Delta x = 5$

Increasing function : L_5 is underest. R_5 is overest.

Lower estimate : $L_5 = f(0)\cdot\Delta x + f(5)\cdot\Delta x + f(10)\cdot\Delta x + f(15)\cdot\Delta x + f(20)\cdot\Delta x$

$= (-42 - 37 - 25 - 6 + 15)\times 5 = -475$

upper estimate . $R_5 = f(5)\cdot\Delta x + f(10)\cdot\Delta x + f(15)\Delta x + f(20)\Delta x + f(25)\Delta x$

$= (-37 - 25 - 6 + 15 + 36)\times 5 = -85$