Section Notes 10/11/07

1 derivative computations

(1) P153, 13.

$f(a) = 3 - 2a + 4a^2$

$f(a+h) = 3 - 2(a+h) + 4(a+h)^2$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{(3-2(a+h)+4(a+h)^2) - (3-2a+4a^2)}{h}$$

$$= \cdots = \lim_{h\to 0} \frac{-2h + 8ah + 4h^2}{h} = \lim_{h\to 0} (-2+8a+4h) = -2+8a$$

(2) P166, 25.

$G(t) = \frac{4t}{t+1}$

$G(t+h) = \frac{4(t+h)}{t+h+1}$

$$G'(t) = \lim_{h\to 0} \frac{G(t+h) - G(t)}{h} = \lim_{h\to 0} \frac{\frac{4(t+h)}{t+h+1} - \frac{4t}{t+1}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{4(t+h)}{t+h+1} - \frac{4t}{t+1}\right) \cdot (t+h+1) \cdot (t+1)}{h \cdot (t+h+1) \cdot (t+1)}$$

$$= \lim_{h\to 0} \frac{4(t+h)(t+1) - 4t(t+h+1)}{h(t+h+1)(t+1)} = \cdots = \lim_{h\to 0} \frac{4h}{h(t+h+1)(t+1)}$$

$$= \lim_{h\to 0} \frac{4}{(t+h+1)(t+1)} = \frac{4}{\lim_{h\to 0}(t+h+1) \cdot \lim_{h\to 0}(t+1)} = \frac{4}{(t+1)^2}$$

domain of the original function: $t \neq 0$ (or $(-\infty, 0) \cup (0, +\infty)$)

domain of the derivative function: $t \neq 0$ (or $(-\infty, 0) \cup (0, +\infty)$)

Remark: In the above examples, you can also use the other formula $f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$ to compute the derivative. The answer you get should be the same as the above one.

(3) P154, 35.

By definition, $f'(0) = \lim\limits_{h\to 0} \dfrac{f(0+h) - f(0)}{h}$ $(h \neq 0)$

$f(0) = 0$ given, $f(0+h) = f(h) = h \sin\frac{1}{h}$ since $h \neq 0$

so $f'(0) = \lim\limits_{h\to 0} \dfrac{h\sin\frac{1}{h} - 0}{h} = \lim\limits_{h\to 0} \sin\frac{1}{h}$

however, this limit doesn't exist since when h approaches 0, the function oscillates between -1 and 1 infinitely many times.

So $f'(0)$ doesn't exist

[2] geometric relation between the original function and the derivative function.

Check list:

original function	derivative function
increasing	positive
decreasing	negative
horizontal tangent	x-intercept
special case: local maximum	cross x-axis from higher to lower
another special case: local minimum	cross x-axis from lower to higher
vertical tangent	infinity discontinuity
linear	horizontal line (constant)
corner	discontinuity

the pair of (original function, derivative function) can be:

(f, f'), or (f', f''), or (antiderivative of f, f)

(Recall: an antiderivative of f is a function F whose derivative is f.

Note: a function f may have infinitely many antiderivatives.)

(1) matching P165. 3.

(a) from left to right:

decreasing → horizontal tangent → increasing → horizontal tangent → decreasing

so its derivative:

negative → 0 → positive → 0 → negative.

so $(a)' = \text{II}$

(b) from left to right:

line segment → corner → line segment → corner → line segment

so its derivative:

horizontal → discontinuity → horizontal → discontinuity → horizontal.

so $(b)' = \text{IV}$

(c) from left to right:

decreasing (when $x<0$) → horizontal tangent ($x=0$) → increasing ($x>0$)

so its derivative

negative (when $x<0$) → 0 ($x=0$) → positive ($x>0$)

So $(c)' = \text{I}$

(d) from left to right:

increasing → hor. tan → decreasing → hor. tan (at $x=0$) → increasing → hor. tan → decreasing

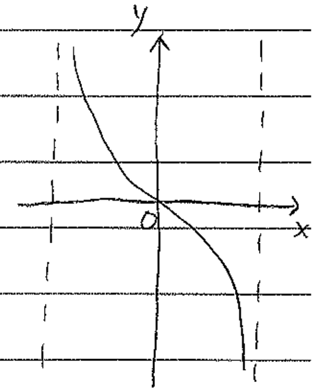
so its derivative:

$>0 \to =0 \to <0 \to =0$ (at $x=0$) $\to >0 \to =0 \to <0$

So $(d)' = \text{III}$

(2) Graph sketching

P166 . 7 ($f \Rightarrow f'$)



from left to right:

vertical tangent → increasing → horizontal tangent (at $x=0$) → decreasing → vertical tangent.

so the derivative:

infinity (+) → positive → 0 (at $x=0$) → negative → infinity (−)

P172 1(c) ($f' \Rightarrow f$)

from left to right, the derivative function:

negative ($0 \to 1$) → 0 (at $x=1$) → positive ($1 \to 5$) → 0 (at $x=5$) → negative ($5 \to 6$)

so the original function:

decreasing ($0 \to 1$) → hor. tan (at $x=1$) → increasing ($1 \to 5$) → hor. tan (at $x=5$) → decreasing ($5 \to 6$)

(3) Graph identification

P167. 37

First observation: at the points where c is decreasing, neither a nor b is negative (or: at the points where c has a horizontal tangent, neither a nor b is equal to 0.)

First conclusion: c cannot be f or f', so $c = f''$

Second observation: when a has a horizontal tangent line, $b=0$; when b has a horizontal tangent line, $c=0$.

Second conclusion: $b=a'$, $c=b'$

Final conclusion: $a=f$, $b=f'$, $c=f''$

P174, 25.

the behavior of f, from left to right

negative $\longrightarrow$ 0 $\longrightarrow$ positive

so an antiderivative should be

decreasing $\longrightarrow$ horizontal tangent $\rightarrow$ increasing

only (b) has the above feature.