Section Notes 09/27/07

§ 1.2. P35.

3. two observations:

(1) odd powers correspond to odd functions.

(2) the higher the power is, the flatter the graph is near 0, and the steeper the graph is far from 0.

7. (the notion of slope).

15. "chirping rate" is related to "temperature"

N	⟷	T
113		70
173		80

(a) want to express T as a linear function of N.

step1. slope $= \frac{80-70}{173-113} = \frac{1}{6}$

step2. function: $T-70 = \frac{1}{6}(N-113) \Rightarrow T = \frac{1}{6}N + \frac{307}{6}$

(OR use other methods to find the y-intercept.)

(b) slope is $\frac{1}{6}$. It means that the temperature in Fahrenheit degrees increases one-sixth as rapidly as the number of cricket chirps per minute. Said differently, each increase of 6 cricket chirps per minute corresponds to an increase of 1°F.

(c) when $N=150$, $T = \frac{150}{6} + \frac{307}{6} \approx 76.2$°F

(NEVER forget the unit when you are doing a real-world application problem!)

§ 1.5. P62

9. start from the exponential function $y = 2^x$

$$y = 2^x \underset{\text{reflection about } y\text{-axis}}{\Longrightarrow} y = 2^{-x} \underset{\text{reflection about } x\text{-axis}}{\Longrightarrow} y = -2^{-x}$$

the graph of $y = 2^x$ can be found in figure 3, page 56

17. you have two points on the graph which give you two pairs of values of (x, y) which satisfy the equation. Write them down and solve the system of equations for C and a. Pay attention that the base of an exponential function should be a positive number, i.e. $a > 0$.

19. the function is $f(x) = 5^x$
so the value of the function at an (abstract) point $x+h$ can be computed by replacing x by $x+h$ in the expression of the function. (think about how you compute $f(2)$.)
Then start from either side of the equation you want to prove:

$$\frac{f(x+h) - f(x)}{h} = \frac{5^{x+h} - 5^x}{h} = \frac{5^x \cdot 5^h - 5^x}{h} = \frac{5^x (5^h - 1)}{h} = 5^x \frac{5^h - 1}{h}$$

25. From the problem, you know: "3 hours" is a "doubling period"

(a) "15 hours" contains 5 "doubling periods", so the size of population is doubled for 5 times. It's $100 \times 2^5 = 3200$

(b) "t hours" contains $\frac{t}{3}$ "doubling periods", so the population is $y = 100 \cdot 2^{\frac{t}{3}}$

(c) let $t = 20$ and compute the value of the function

(d) let $y = 50000$ and solve for t

[About proofs]

§ 1.3. P48

63. I only prove one statement here, you should do the rest three by yourself by modifying my proof here a little bit.

Given: f, g are even functions, Prove: $f+g$ is an even function.

Proof. Since f is even, $f(-x) = f(x)$ for all x in the domain.

Since g is even, $g(-x) = g(x)$.

Add the above two equations together, we have

$$f(-x) + g(-x) = f(x) + g(x)$$

which is the same as

$$(f+g)(-x) = (f+g)(x)$$

By definition, $f+g$ is an even function.

§ 1.5 P63

31. Proof: Since $f(x) = \dfrac{1-e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$, we have $f(-x) = \dfrac{1-e^{\frac{1}{-x}}}{1+e^{\frac{1}{-x}}}$

However, $\dfrac{1-e^{\frac{1}{-x}}}{1+e^{\frac{1}{-x}}} = \dfrac{1-e^{-\frac{1}{x}}}{1+e^{-\frac{1}{x}}} = \dfrac{1-\frac{1}{e^{\frac{1}{x}}}}{1+\frac{1}{e^{\frac{1}{x}}}} = \dfrac{e^{\frac{1}{x}}\left(1-\frac{1}{e^{\frac{1}{x}}}\right)}{e^{\frac{1}{x}}\left(1+\frac{1}{e^{\frac{1}{x}}}\right)}$ (multiply by the same factor)

$$= \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} = \frac{-(1-e^{\frac{1}{x}})}{1+e^{\frac{1}{x}}} = -\frac{1-e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$$

This gives: $f(-x) = -f(x)$

So $f(x)$ is an odd function.

(Think about it and come to me if you are confused.)