Section Notes 09/25/07

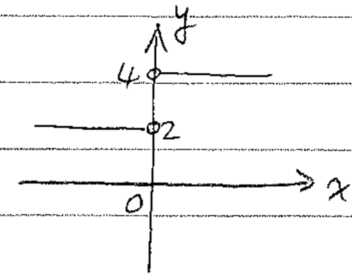
Warm up questions. P22 §1.1 1, 5, 7

(In problem 7, pay attention to 2 different kinds of endpoints.)

39. (example of dealing with absolute values)

domain: $(-\infty, 0) \cup (0, +\infty)$

two cases: $G(x) = \begin{cases} \frac{3x+x}{x} = 4 & \text{if } x > 0 \\ \frac{3x-x}{x} = 2 & \text{if } x < 0 \end{cases}$



57. (a function together with its domain)

$V = (20-2x)(12-2x)x = 4x^3 - 64x^2 + 240x$

restrictions to x: $\begin{cases} 20-2x > 0 \\ 12-2x > 0 \\ x > 0 \end{cases} \Rightarrow \begin{cases} x < 10 \\ x < 6 \\ x > 0 \end{cases} \Rightarrow 0 < x < 6$

domain: $(0, 6)$

(I will summarize the typical procedure of computing domains at a later time.)

61. (easy, geometric picture of symmetries)

P46 §1.3

19. essential step: completing the square

for a general quadratic polynomial, $(a \neq 0)$

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$$

$$= a\left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

procedure:

Step1: factor out the coefficient of x^2

Step2: add and subtract the square of half of the coefficient of x

Step3: Complete the square and compute the new constant term.

In this problem:

$$y = \frac{1}{2}(x^2+8x) = \frac{1}{2}(x^2+8x+16-16) = \frac{1}{2}\left((x+4)^2-16\right)$$

the route map from $y=x^2$ to $y=\frac{1}{2}(x^2+8x)$:

$$y=x^2 \underset{\text{(shift left)}}{\Longrightarrow} y=(x+4)^2 \underset{\text{(shift down)}}{\Longrightarrow} y=(x+4)^2-16 \underset{\text{(compress)}}{\Longrightarrow} y=\frac{1}{2}\left((x+4)^2-16\right)$$

another example of completing a square:

$$y = 2x^2 - x + 4$$

Step 1: $y = 2(x^2 - \frac{1}{2}x + 2)$

Step 2: $y = 2(x^2 - \frac{1}{2}x + (\frac{1}{4})^2 - (\frac{1}{4})^2 + 2)$

Step 3: $y = 2((x-\frac{1}{4})^2 + \frac{31}{16})$

route map:

$$y=x^2 \underset{\text{(shift right)}}{\Longrightarrow} y=(x-\tfrac{1}{4})^2 \underset{\text{(shift up)}}{\Longrightarrow} y=(x-\tfrac{1}{4})^2+\tfrac{31}{16} \underset{\text{(stretch)}}{\Longrightarrow} y=2\left((x-\tfrac{1}{4})^2+\tfrac{31}{16}\right)$$

a few more similar exercises:

(1) x^2-16x

(2) y^2+7y

(3) x^2-6x+1

(4) $2x^2+6x+7$

(5) $3t^2-2t-1$

31 (nothing fancy, introduce the interval notation of expressing domains)

47 (composition of functions, better do it than not)

51 (Just do (a), (d), easy but be careful ...)