INTERGRAL CALCULUS CHECKLIST

Contents

1. Integral Computation

There are three majoy tools: evaluation theorem, substitution, integration by parts. They should be tried in the above order, namely, if it is very easy to obtain the antiderivative, use evaluation theorem, otherwise, try substitution before integration by parts. The integral computation is actually a lot more tricky than it looks like. And you can learn how to apply these techniques only in practice. However, remember our LIATE or DETAIL principle in integration by parts. In some problems, you may need to apply it for several times.

You do need a lot of practice on this part, because it will take a large portion in the final (last year it was 24 points out of 150). There are lots of exercises in textbook, do as many as possible, and pay attention to those listed in daily homework and the 10th uncollected weekly homework.

Sample Problems: problem 1-28 on page 374, problem 7-54 on page 392, problem 3-28 on page 398.

2. Estimate a Definite Integral via Approximating Rectangles

There are graph-type problems and table-type problems. In the graph-type problems, according to the given function defined on a closed interval, and the number of small intervals that you separate the whole interval into, you should be able to draw the approximating rectangles and use them to estimate the definite integral. There are three methods: left endpoint rule, right endpoint rule and midpoint rule. You should be able to tell whether your

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estimate is an overestimate or an underestimate, provided the given function is increasing or decreasing.

In table-type problems you are not given the whole function but instead you are given the values of the function at a couple of points. This is already enough information. Use the difference between the points as the width of rectangles and the values of the function as the height of rectangles. Multiply them and compute the sum.

Sample Problems: problem 1-4 on page 352, problem 11-16 on page 353, problem 5-12 on page 364.

3. Evaluate a Definite Integral via Limit of Riemann Sums

I know that this is messy. So, just focus on the case in which the integrand is a polynomial. You should remember the formula of Riemann sums, which is actually very intuitive. The steps of doing these problems are:

(1) Compute the size of small intervals Δx and the coordinates of the separating points x_i, which involves the number of intervals n.
(2) Use the formula to write the Riemann sum as a summation. For right endpoints, the formula is $\sum_{i=1}^{n} f(x_i)\Delta x$. Refer to the textbook for the left endpoint formula.
(3) Simplify the Riemann sum and use the given formula sheet to compute the summation. Remember, n is always considered as a constant in this step.
(4) Let n approach ∞ and compute the limit.

Sample Problems: problem 21-25 on page 365.

4. Evaluate a Definite Integral by Computing Net Area

If the graph of the function is given, simply compute the net area between the graph of the function and the x-axis. Remember that area above x-axis is positive and area below x-axis is negative. If the graph is not explicitly given, draw it by yourself. The shape of the graph should be something very familiar.

Sample Problems: problem 31-38 on page 365.

5. First Part of Fundamental Theorem

If you see an integral with variable upper limit like $g(x) = \int_a^x f(x)dx$ in whichever problem, where a is a constant while $f(x)$ is a continuous function, you should remind yourself that $f(x)$ is the derivative of $g(x)$ and you should be ready to apply FTC1. This could happen in a large variety of situations in which you are asked to compute the derivative of $g(x)$, such as derivative computation, L'Hopital's rule, intervals of increase or decrease, etc. So keep yourself sensitive to integrals with variable upper limits.

If the upper limit is a function of the independent variable, then you need to apply chain rule. If it is the lower limit instead of the upper limit which is variable, exchange the two limits and adjust the sign. If both of them are functions, break up the integral into the sum of two at a constant number.

Sample Problems: problem 2-20 on page 383.

6. Application Problems

There are two types of application problems. In the first type, you should be able to explain the meaning of a specific definite integral under a real-world situation. You should make your interpretation kind of complete and rigorous.

In the second type of problems, given the velocity function, you are asked to compute displacement and distance traveled. The displacement is exactly the integral of the velocity function, or geometrically, the net area between the graph of velocity and x-axis. And the distance is the integral of the absolute value of velocity function, or geometrically, the absolute area between the graph and the x-axis, where you use positive sign for both areas above and below the x-axis.

Sample Problems: problem 47-58 on page 375

7. Verify an Indefinite Integral

We had only one problem from the homework, while this type of problems appeared again and again in finals of recent years. If you are given the result of an indefinite integral, then simply compute the derivative of the result, which should be the integrand of the original integral. That's it!

Sample Problems: problem 37-38 on page 374

8. Piecewise Functions

Principally it is not too much different from a function which is given by a single formula, so don't get frustrated when you see a piecewise function. All techniques are the same as above. When you are confused, try to draw a graph to help you think geometrically, which is typically easier. The integral of an absolute value can be considered as a special case of this type. There are quite a few problems in the old practice finals. Try to work on them a little bit.

Sample Problems: problem 2,3 on page 383, problem 26 on page 384

9. Examples

(1) Evaluate $\displaystyle\int x^5 \cos x^3 dx$.

(2) Evaluate $\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \tan^3 \theta d\theta$.

(3) A particle is moving along a line. The acceleration function (in m/s^2) is given by $a(t) = 2t + 3$ and the initial velocity (in m/s) $v(0) = -4$. Find the displacement and the distance traveled during the time interval $0 \leqslant t \leqslant 3$.

(4) Use the limit of a Riemann sum to evaluate the integral $\displaystyle\int_{-1}^{3} x^2 dx$.

(5) Differentiate the function $y = \displaystyle\int_{\sin x}^{\cos x} (1 + v^2)^{10} dv$.

(6) A right circular cone is inscribed in a sphere of radius R. Find the largest possible volume of such a cone.

(7) A 15-foot ladder is resting against the wall. The bottom is initially 10 feet away from the wall and is being pushed towards the wall at a rate of $\frac{1}{4}$ ft/sec. How fast is the top of the ladder moving up the wall 12 seconds after we start pushing?

10. Final Remarks

I guess these are all that come up in my mind. If you don't have enough time, try to work on one or two examples of each type from the practice finals, and then focus on your weakest part. Also spend some time on the stuffs covered in the two midterms, especially those types of problems you couldn't do at the two midterms.

Questions are always welcomed. Refer to the course webpage for the most updated office hours. Thank you for attending my sections this quarter. And best luck to your finals.