Domains and Symmetries*

- ## Tips for finding the domain of a function

Basically, if the domain of a function is not stated explicitly, it is by default the set of all numbers for which the formula makes sense and defines a real number.

In order to find the domain of a given function, try to find out all restrictions to the independent variable (usually x).

Familiar Restrictions to the Independent Variable	
If you see ...	**Restriction**
$h(x)$ in a denominator	$h(x) \neq 0$
$\sqrt[n]{h(x)}$ where n is even	$h(x) \geqslant 0$
$\log h(x)$	$h(x) > 0$
$\tan h(x)$	$h(x) \neq k\pi + \frac{\pi}{2}$ where k is an integer
$\cot h(x)$	$h(x) \neq k\pi$ where k is an integer
some other functions	corresponding restrictions

Here in this table, by $h(x)$ I mean any expression (appearing in the function) about x.

Moreover, if the problem is a real-world problem, you should also consider restrictions from real world. For example, the length, width and height of a box should be strictly positive.

*by Ziyu Zhang, Sept 29, 2005; revised on Oct 2, 2007

Finally, if you didn't find any restrictions to the independent variable, the domain of the function is **R** (i.e. all real numbers). For example, if no real-world restrictions applied, the domains of all polynomials, sine/cosine functions and exponential functions should be **R**, .

There are many exercises of this topic in the textbook. You can refer to example 7 (page 17), exercise 27-31 (page 23) and exercise 57 (page 24) of section 1.1, and a lot more in section 1.6.

- ## Descriptions of symmetries of functions

There are two ways to describe the symmetries of functions, namely, the geometric way and the algebraic way.

Descriptions of Symmetries of Functions

	Geometric description	**Algebraic description**
Even function	the graph is symmetric with respect to the y-axis	$f(-x) = f(x)$
Odd function	the graph is symmetric about the origin	$f(-x) = -f(x)$
Advantage	Very intuitive	Rigorous and clear in a proof

So, basically, if we are given a graph of a function, it is very direct to determine the symmetry of the function by the geometric description. However, if the graph is not given, or if we want to rigorously prove a certain function is even or odd, the algebraic description works better in most cases.

Finally, please note that there are a lot of functions which are neither even nor odd, and there are some functions which are both even and odd as well.

For more examples, please refer to example 12 of section 1.1 (page 20), exercise 65-70 of section 1.2 (page 25), exercise 63 of section 1.3 (page 48) and exercise 31 of section 1.6 (page 63).