A Mathematical Introduction to Data Science **April 26, 2016**

Homework 8. Generalized LLE in Manifold Learning

Instructor: Yuan Yao *Due: Tuesday May 3, 2016*

The problem below marked by * is optional with bonus credits.

1. *Manifold Learning*: The following codes by Todd Wittman contain major manifold learning algorithms talked on class.

   `http://math.stanford.edu/~yuany/course/data/mani.m`

   Precisely, eight algorithms are implemented in the codes: MDS, PCA, ISOMAP, LLE, Hessian Eigenmap, Laplacian Eigenmap, Diffusion Map, and LTSA. The following nine examples are given to compare these methods,

   (a) Swiss roll;
   (b) Swiss hole;
   (c) Corner Planes;
   (d) Punctured Sphere;
   (e) Twin Peaks;
   (f) 3D Clusters;
   (g) Toroidal Helix;
   (h) Gaussian;
   (i) Occluded Disks.

   Run the codes for each of the nine examples, and analyze the phenomena you observed.

   * Moreover if possible, make an implementation of Vector Diffusion Map (Laplacian) to reconstruct Swiss roll etc., and compare it against others. Note that LTSA looks for the following coordinates,

   $$\min_Y \sum_{i\sim j} \|y_i - U_i U_j^T y_j\|^2$$

   where U_i is a local PCA basis for tangent space at point $x_i \in \mathbb{R}^p$; in a contrast, vector connection Laplacian looks for:

   $$\min_Y \sum_{i\sim j} \|y_i - O_{ij} y_j\|^2, \quad O_{ij} = \arg\min_O \|U_i - O_{ij} U_j\|^2$$

   where O_{ij} is an isometry (parallel transportation) from the tangent space at point x_j to the tangent space at point x_i, whose solution is given by the lower eigenvectors of the following vector connection Laplacian

   $$L_{ij} = \begin{cases} -w_{ij} O_{ij}, & i \neq j \\ \sum_j w_{ij} I, & i = j. \end{cases}$$

Here $w_{ij} = w_{ji}$ defines the similarity (neighborhood) between point x_i and x_j. For details, please refer to the paper by Amit Singer and Hau-Tieng Wu, *Vector diffusion maps and the connection laplacian*, Comm. Pure Appl. Math. 65 (2012), no. 8, 10671144. You may try Hau-Tieng Wu's matlab codes on VDM from the following website

`https://sites.google.com/site/hautiengwu/home/download`