Math 52- Winter 2010 - Midterm Exam I

Please circle your TA’s name: Jack Hall  Xiannan Li  David Sher
Circle the time your TTh section meets: 10:00  11:00  1:15  2:15
Your name (print):  
Student ID:  

Please sign the following:
"On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

Signature: ________________________________

Instructions:
• Print your name and student ID number, circle your TA’s name, the time that you attend the TTh section and sign to indicate that you accept the Honor Code.
• Read each question carefully. In order to receive full credit, please show all of your work and justify your answers.
• You do not need to simplify your answers unless specifically instructed to do so. You may use any result from class that you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
• You have 2 hours. This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.
• If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
• It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until Thursday, Feb 11, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.

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Formulas you may use: Relating rectangular coordinates \(x, y, z\), cylindrical coordinates \(r, \theta, z\) and spherical coordinates \(\rho, \theta, \varphi\):

\[
\begin{align*}
  z &= \rho \cos \varphi, \\
  r &= \rho \sin \varphi, \\
  x &= r \cos \theta = \rho \sin \varphi \cos \theta, \\
  y &= r \sin \theta = \rho \sin \varphi \sin \theta
\end{align*}
\]

and \(dxdydz = rdrd\theta dz = \rho^2 \sin \varphi d\rho d\theta d\varphi\).

Trig identities: \(2 \cos^2 x = 1 + \cos 2x\), \(2 \sin^2 x = 1 - \cos 2x\).
Problem 1. (14pts) Change the order of integration in:

$$\int_{-1}^{2} \int_{0}^{x^2} \sin^2(x + 3y) \, dy \, dx$$

DO NOT EVALUATE THE INTEGRAL!
Problem 2. (14pts) Compute

\[ \int_0^1 \int_y^{\sqrt{y}} e^{x^2} \, dx \, dy \]
Problem 3. (14pts) Find the area of the planar region $R$ which is situated in the first quadrant, enclosed by $r = 2 \cos \theta$ and outside the circle $x^2 + y^2 = 1$. 
Problem 4. (14pts) Find the volume of the region in the first octant bounded inside both cylinders $x^2 + y^2 \leq 1$ and $x^2 + z^2 \leq 1$. 
Problem 5. (14pts) Find the coordinates of the centroid of the solid bounded from above by the cone
\[ z = \sqrt{x^2 + y^2}, \]
below by the \( z = 0 \) and on the sides by the cylinder \( x^2 + y^2 = 4 \). (It is part of a cylinder
hollowed out by a cone).
Problem 6. (16pts total) Consider the thick bowl $B$ which is the lower half of the region between the spheres (centered at the origin) of radius 1 and 2 and whose density is $\delta(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

(a) (10 points) Find the total mass of $B$.

(b) (6 points) Setup, BUT DO NOT EVALUATE an explicit ITERATED integral representing the moment of inertia of $B$ about the $x$-axis.
Problem 7. (14pts) Consider the region $R$ (in the first quadrant) bounded by the curves $x = y$, $x = 2y$, $xy = 1$ and $xy = 2$. Calculate

$$\iint_R \left( \frac{x}{y} \right)^{100} \cdot \cos(xy) \, dxdy$$

**Hint:** Use an appropriate change of variables.