This is a closed-book, closed-notes exam. No calculators or other electronic devices will be permitted. **You have 2 hours.** If you finish early, you must hand your exam paper to a member of teaching staff.

In order to receive full credit, please show all of your work and justify your answer. You do not need to simplify your answers unless specifically instructed to do so.

If you need extra room, use the back sides of each page. There is also a blank page at the end of the exam for you to use. Do not unstaple or detach pages from this exam.

Please sign the following:

> “On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

**Signature: ____________________________**
1. (6 points)

Sketch the region of integration $R$ in the $xy$-plane for the integral below. Then reverse
the order of integration; i.e., write an iterated integral with explicit limits in which the $y$-
inTEGRATION IS DONE first. NO EVALUATION OF INTEGRALS IS REQUIRED.

$$
\int \int_{R} g(x, y) \, dx \, dy = \int_{1}^{\sqrt{5-y}} \int_{0}^{5} g(x, y) \, dx \, dy
$$
2. (8 points)

Below is an integral over a solid region in $xyz$-space. Rewrite it as an iterated integral in spherical co-ordinates over the same region. Evaluate the integral which is written in spherical co-ordinates.

\[
\int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} \int_{1}^{\sqrt{4-(x^2+y^2)}} \frac{4}{\sqrt{x^2+y^2+z^2}} \, dz \, dy \, dx
\]
3. (10 points)
Consider the circle which is described in polar coordinates by \( r = 2 \sin \theta \). Let \( D \) be a plate consisting of points inside this circle and outside the circle \( r = 1 \). Suppose the density \( \delta(x, y) \) at a point \((x, y)\) equals \( y \).

a) Set up a double integral in polar co-ordinates, with explicit limits of integration, for the mass of the plate. DO NOT EVALUATE THE INTEGRAL.

b) Set up a formula for the \( y \)-coordinate \( \bar{y} \) of the center of mass of the plate in terms of integrals in polar coordinates with explicit limits of integration. DO NOT COMPUTE the value for \( \bar{y} \). Explain why you know, without computation, that the \( x \)-coordinate for the center of mass is 0.
4. (10 points)

Consider the cone in $\mathbb{R}^3$ determined by the condition that the spherical coordinate $\phi$ (angle with positive $z$-axis) satisfies $0 \leq \phi \leq \frac{\pi}{3}$. Let $W$ be the portion of this cone which lies between the sphere of radius 2 and the sphere of radius 1.

a) Compute the volume of $W$.

b) Find the average distance of a point in $W$ to the origin.
5. (8 points)

Consider the triangular region $D$ in the $(x, z)$-plane bounded by the lines $x = 2$, $z = 1$, and $x = z$. Let $W$ be the 3-dimensional solid obtained by rotating $D$ all the way around the $z$-axis. (W is like a donut with a triangular cross-section.) Set up an iterated integral in cylindrical coordinates that computes the volume of $W$ and then evaluate that integral.
6. (10 points)

Let $D$ be the diamond-shaped region in the $xy$ plane with vertices at $(0,0), (\pi, \pi), (0, 2\pi), (-\pi, \pi)$. Consider the transformation $T$ from the $uv$ plane to the $xy$ plane defined by

$$T(u, v) = \left( \frac{u - v}{2}, \frac{u + v}{2} \right) = (x, y).$$

a) Draw a sketch of $D$ in the $xy$ plane and find the rectangle $C$ in the $uv$ plane which is taken by $T$ to $D$.

b) Set up an integral (using the transformation $T$ as a change of coordinates) over the region $C$ in the $uv$-plane that can be used to compute the following integral over $D$:

$$\int \int_D (x - y)^2 \sin^2(x + y) \, dx \, dy.$$ 

Then COMPUTE the integral over $C$. 
7. (8 points)
   a) Set up an iterated integral that computes the integral of $f(x, y, z) = x$ over the following region in $\mathbb{R}^3$:
      
      \[ 0 \leq x, \ 0 \leq y, \ x + y \leq 1, \ 0 \leq z \leq x + y. \]

      Integrate first in the $z$-direction and then the other two directions in either order. Do NOT evaluate the integral.

   b) Sketch the region in part a). Set up the limits of integration in order to repeat part a), this time integrating first in the $y$-direction. Do NOT compute the integral(s).