Math 52 - Winter 2007 - Midterm Exam I

Name: __________________________

Student ID: ______________________

Section number and TA name: ___________

Signature: __________________________

Instructions: Print your name and student ID number, print your section number and TA’s name, write your signature to indicate that you accept the honor code. During the test, you may not use notes, books, calculators. Read each question carefully, and show all your work.
There are nine problems on the pages numbered from 1 to 9, with the total of 100 points. Point values are given in parentheses. You have 2 hours (until 9PM) to answer all the questions.

<table>
<thead>
<tr>
<th>Question</th>
<th>Score</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
Problem 1. (8 pts.) Let $R$ be a region of $\mathbb{R}^2$ symmetrical about the $y$-axis, and $R_+$ be the “half” of the region $R$ contained in the half plane $x \geq 0$. Mark as TRUE/FALSE the following statements:

a) $\int \int_R dA = 2 \cdot \int \int_{R_+} dA$  \hspace{1cm} TRUE \hspace{1cm} FALSE

b) $\int \int_R x \, dA = 2 \cdot \int \int_{R_+} x \, dA$  \hspace{1cm} TRUE \hspace{1cm} FALSE

c) $\int \int_R \sin(y) \, dA = 2 \cdot \int \int_{R_+} \sin(y) \, dA$  \hspace{1cm} TRUE \hspace{1cm} FALSE

d) $\int \int_R x^2 \, dA = 2 \cdot \int \int_{R_+} x^2 \, dA$  \hspace{1cm} TRUE \hspace{1cm} FALSE
Problem 2. (10 pts.) Setup, but do not evaluate the triple iterated integral representing the volume of the bounded solid \( W \) bounded by the graphs of the paraboloid \( x^2 + y^2 = z \) and the plane \( z = 2x + 3 \).
Problem 3. (12 pts.) Evaluate

\[ \int_{0}^{1} \int_{1}^{3} y \cdot \cos (x^3) \, dx \, dy + \int_{1}^{3} \int_{y}^{3} y \cdot \cos (x^3) \, dx \, dy \]
Problem 4. (12 pts.) Change the order of integration in

\[ \int_{-1}^{2} \int_{0}^{x^2} \sin(x + 3y) \, dy \, dx \]

DO NOT EVALUATE THE INTEGRAL
Problem 5.

1. (4 pts.) Sketch the region in the $xy$-plane described by the inequality

$$|x| + |y| \leq 1$$

2. (6 pts.) Show that

$$\int\int_{|x|+|y|\leq 1} f(x+y) \, dx \, dy = \int_{-1}^{1} f(t) \, dt,$$

for any continuous real-valued function $f$.

**Hint:** Perform some change of variables...
Problem 6. (12 pts.) Consider the region $W$ on $xyz$ space defined by inequalities: $1 \leq x \leq 2, 0 \leq xy \leq 2, 0 \leq z \leq 1$.

a) Find the Jacobian $\frac{\partial (x, y, z)}{\partial (u, v, w)}$, where $u = x, v = xy$ and $w = z$.

b) Evaluate the integral

$$\int \int \int_W (x^2y + 3xyz) \, dx \, dy \, dz$$

by applying the above change of coordinates.
Problem 7. (12 pts.) Find the coordinates of the centroid of the solid in the first octant that is bounded from above by the cone \( z = \sqrt{x^2 + y^2} \), below by the \( z = 0 \) and on the sides by the cylinder \( x^2 + y^2 = 4 \). (It is part of a cylinder hollowed out by a cone)

**Hint 1:** use cylindrical coordinates.

**Use:** The volume of a cylinder with radius of the base \( r \) and the height \( h \) is \( \pi r^2 h \). The volume of a the cone with radius of the base \( r \) and the height \( h \) is \( \frac{1}{3} \pi r^2 h \). Use these to find the total mass of your solid.
Problem 8. (12 pts.) Let $S$ be the part of the ball $x^2 + y^2 + z^2 \leq r^2$ that lies above the $xy$ plane. Find the moment of inertia of $S$ about the $z$ axis.

Hint: use spherical coordinates.

Use: $\int_0^{\pi/2} \sin^3 x \, dx = \frac{2}{3}$. 
Problem 9. (12 pts.) Assume that a region $R$ with density $\rho \equiv 1$ in the $xy$ coordinates is symmetrical about the $x$ axis. Show that

$$I_{(0,b)} = I_{(0,0)} + m \cdot b^2$$

where $I_{(a,b)}$ is the moment of inertia about an axis perpendicular to $xy$ plane at the point $(a,b)$ and $m$ is the total mass of the region.