Math 52 - Winter 2006 - Midterm Exam I

Name: ________________________________

Student ID: __________________________

Section number and TA name: __________

Signature: _____________________________

**Instructions:** Print your name and student ID number, print your section number and TA’s name, write your signature to indicate that you accept the honor code. During the test, you may not use notes, books, calculators. Read each question carefully, and show all your work.

There are nine problems on the pages numbered from 1 to 9, with the total of 100 points. Point values are given in parentheses. You have 2 hours (until 9PM) to answer all the questions.

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Problem 1. (8 pts.) Let $R$ be a region of $\mathbb{R}^2$ symmetrical about the $y$-axis, and $R_+$ be the “half” of the region $R$ contained in the half plane $x \geq 0$. Mark as TRUE/FALSE the following statements:

a) \( \int \int_R dA = 2 \cdot \int \int_{R_+} dA \)  
   \hspace{1cm} TRUE \hspace{1cm} FALSE

b) \( \int \int_R x \, dA = 2 \cdot \int \int_{R_+} x \, dA \)  
   \hspace{1cm} TRUE \hspace{1cm} FALSE

c) \( \int \int_R \sin(y) \, dA = 2 \cdot \int \int_{R_+} \sin(y) \, dA \)  
   \hspace{1cm} TRUE \hspace{1cm} FALSE

d) \( \int \int_R x^2 \, dA = 2 \cdot \int \int_{R_+} x^2 \, dA \)  
   \hspace{1cm} TRUE \hspace{1cm} FALSE
Problem 2. (12 pts.) Evaluate the integral $\iiint_T x^2 \, dV$, where $T$ is the tetrahedron bounded by coordinate planes and the plane $2x + 3y + z = 6$. 
Problem 3. (12 pts.) Setup, but do not evaluate the triple integral representing the volume of the bounded solid $W$ bounded by the graphs of the cylinders $x^2 + z^2 = 1$, $y^2 + z^2 = 1$ that is contained the first octant of the coordinate system.
Problem 4. (12 pts.) Evaluate

\[
\int_1^e \int_{\ln y}^1 \frac{1}{y} \cdot e^{x^2} \, dx \, dy
\]
Problem 5. (12 pts.) Change the order of integration in

\[ \int_{-1}^{2} \int_{0}^{\vert x \vert} \sin(x + 3y) \, dy \, dx \]

DO NOT EVALUATE THE INTEGRAL
Problem 6.

1. (4 pts.) Sketch the region in the $xy$–plane described by the inequality

$$|x| + |y| \leq 1$$

2. (6 pts.) Show that

$$\int \int \frac{f(x + y) \, dx \, dy}{|x| + |y| \leq 1} = \int_{-1}^{1} f(t) \, dt,$$

for any continuous real-valued function $f$.

**Hint:** Perform some change of variables...
Problem 7. (10 pts.) Consider the change of variables \( u = xy, v = yz, w = xz \).

a) Find the Jacobian \( \frac{\partial (x, y, z)}{\partial (u, v, w)} \).

b) Find the volume of the region in the first octant enclosed by the hyperbolic cylinders \( xy = 1, xy = 4, xz = 1, xz = 4, yz = 4, yz = 9 \).

Hint: Use the fact that \( uvw = x^2y^2z^2 \).
Problem 8. a) (8 pts.) Set up an integral in **cylindrical coordinates** that represents the volume of the region in $\mathbb{R}^3$ bounded by the two surfaces $z = x^2$ and $z = 9 - y^2$.

b) (4 pts.) evaluate the integral in part a).
Problem 9. (12 pts.) Let $S$ be the part of the ball $x^2 + y^2 + z^2 \leq 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$. Find the volume of $S$.

Hint: use spherical coordinates.