Math 52 - Fall 2007 - Midterm I

Name: ________________________________

Student ID: ____________________________

Signature: ______________________________

Instructions:

Please print your name and student ID. Your signature indicates that you accept the honor code. During the test, you may not use notes, books, calculators or telephones. Read each question carefully, and show all your work. There are 8 questions; each question is worth 10 points.

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Problem 1.

Change the order of integration and evaluate

$$\int_{0}^{1} \int_{\sqrt{x}}^{1} \cos(y^3) \, dy \, dx.$$
Problem 2.

Find the area of the region between by the curves $xy = 1$, $y = x^2$ and $y^2 = 8x$. 
Problem 3.

Compute

\[ \iint_D \frac{x + y}{(x - y + 1)^2} \, dx \, dy, \]

where \( D \) is the triangle with vertices \((0, 0), (\frac{1}{2}, \frac{1}{2})\) and \((1, 0)\).
Problem 4.

Consider a plate bounded by the curves $xy = 1$, $xy = 2$, $xy^2 = 1$ and $xy^2 = 4$, and assume that the density function is $\delta(x, y) = y$. Find the first coordinate $\bar{x}$ of the center of mass.
Problem 5.

Find the polar moment of inertia of a plate bounded by the lines \( x = 1, x = 2, y = 0 \) and \( y = x \), assuming that the density function is

\[ \delta = (x^2 + y^2)^{-\frac{5}{2}}. \]

You may want to use polar coordinates.
Problem 6.

Consider the field \( \mathbf{F} = (x + y) \mathbf{i} + 4y \mathbf{j} \).

Compute the integral \( \int_C \mathbf{F} \cdot ds \),

taking the curve \( C \) to be

(i) the line segment starting at \((1, 1)\) and ending at \((-1, 0)\);  
(ii) the arc of the parabola \(2y^2 = x + 1\) starting at \((1, 1)\) and ending at \((-1, 0)\).

Is the field \( \mathbf{F} \) a gradient field?
Problem 7.

(i) Consider the field \( F = i - j \). How should you place a unit line segment \( C \) so that

\[
\int_C F \cdot ds = 1
\]

(ii) Consider the field \( F = \nabla f \), where

\[
f(x, y) = x^2 y - x y^3 + y^2.
\]

Let \( C \) be any curve starting at \((1, 1)\) and ending somewhere on the \( x \)-axis. Show that the line integral

\[
\int_C F \cdot ds
\]

is independent of the path \( C \), and compute its value.
Problem 8.

Let \( f(x) \) and \( g(y) \) be two continuously differentiable functions defined everywhere on the real line. Consider the field

\[ F = (f(x) + y^2) \mathbf{i} + (g(y) + xy) \mathbf{j}. \]

(i) Starting from the definitions, show that

\[ \int_C F \cdot ds = 0, \]

where \( C \) is the square of side 1 with corners at \((-1, -1), (1, -1), (1, 1) \) and \((-1, 1)\).
(ii) Can it possibly be true that

\[ \int_C \mathbf{F} \cdot ds = 0 \]

for any simple closed curve \(C\)? Please justify your answer.