Math 52 - Autumn 2006 - Midterm Exam I

Name: ________________________________

Student ID: __________________________

Signature: ____________________________

Instructions: Print your name and student ID number, write your signature to indicate that you accept the honor code. During the test, you may not use notes, books, calculators. Read each question carefully, and show all your work.
There are five problems with the total of 100 points. Point values are given in parentheses. You have 50 minutes to answer all the questions.

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Problem 1.

a) (10 points) Sketch the region of integration of \( \int_0^2 \int_y^1 \cos(x^2) \, dx \, dy \)

b) (10 points) Evaluate \( \int_0^2 \int_y^1 \cos(x^2) \, dx \, dy \)
Problem 2. (20 points) Let $W$ be the solid defined by the inequalities:

$$\begin{align*}
  z &\leq 1 - y^2 \\
  z &\geq y^2 - 1 \\
  x + z &\leq 1 \\
  x &\geq 0
\end{align*}$$

For the above solid $W$ write $\iiint_W f(x, y, z)\,dV$ as a triple iterated integral over an $x-$simple region. (there are still two possible choices of the order of the integrals, so just choose one of them...)
Problem 3. (20 points) Find the moment of inertia with respect to the line \( x = 1 \) of a thin sheet of constant density \( \rho = 1 \) bounded by the curve \( y = \left( \frac{\sin^2 x}{(x - 1)^2} \right) \), the \( x \)-axis and the strip \( \pi \leq x \leq 2\pi \).

You may use the formula:

\[
\int_{\pi}^{2\pi} \sin^2 x \, dx = \frac{\pi}{2}
\]
Problem 4. (20 points) Consider the change of variables $u = xy$, $v = yz$, $w = xz$.

a) Find the Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

**Hint:** Use the fact that $uvw = x^2y^2z^2$.

b) Find the volume of the region in the first octant enclosed by the hyperbolic cylinders $xy = 1$, $xy = 4$, $xz = 1$, $xz = 4$, $yz = 4$, $yz = 9$. 
Problem 5. (20 points) Evaluate the integral:

$$\int_{-a}^{a} \int_{0}^{\sqrt{a^2 - y^2}} e^{x^2 + y^2} \, dx \, dy$$

Hint: Use polar coordinates.