Math 52 - Autumn 2005 - Midterm Exam I

Name: _________________________________

Student ID: ____________________________

Signature: ______________________________

**Instructions:** Print your name and student ID number, write your signature to indicate that you accept the honor code. During the test, you may not use notes, books, calculators. Read each question carefully, and show all your work.
There are five problems with the total of 100 points. Point values are given in parentheses. You have 50 minutes to answer all the questions.

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Problem 1.

a) (10 points) Sketch the region of integration of $\int_0^1 \int_y^1 e^{-x^2} \, dx \, dy$

b) (10 points) Evaluate $\int_0^1 \int_y^1 e^{-x^2} \, dx \, dy$
Problem 2. Let $W$ be the solid bounded by the graphs:

\[
\begin{align*}
  z &= 1 - y^2 \\
  z &= y^2 - 1 \\
  x + z &= 1 \\
  x &= 0
\end{align*}
\]

a) (10 points) In the descriptions of the solid $W$ change all equations into inequalities.

b) (10 points) For the above solid $W$ write \[\int \int \int_{W} f(x, y, z) \, dV\] as a triple iterated integral over an $x$–simple region. (there are still two possible choices of the order of the integrals, so just choose one of them...)
Problem 3. (20 points) Let $W$ be a solid, $l$ any line and $l_0$ a line parallel to $l$ passing through the center of mass of $W$. Show that

$$I_l = I_{l_0} + M \cdot d^2$$

where $I_l$ is the moment of inertia of $W$ about the line $l$, $M$ is the mass of $W$ and $d$ is the distance between the line $l$ and $l_0$.

**Hint 1:** By rotating the whole solid we can assume that both lines are parallel to the $z-$axis and that the line $l$ is the $z-$axis. Then start the computations from writing the formula for $I_{l_0}$.

**Hint 2:** In the above position of the lines $l$ and $l_0$, if $(x_0, y_0, z_0)$ denotes the coordinates of the center of mass of $W$, then $d^2 = x_0^2 + y_0^2$. 
Problem 4. (20 points) Let $a$ and $b$ be any numbers such that $a^2 + b^2 = 1$ and $f$ be a continuous function of one variable. Perform the change of variables

\[
\begin{cases}
  u = ax + by \\
  v = bx - ay
\end{cases}
\]

to show that

\[
\iint_{x^2 + y^2 \leq 1} f(ax + by) \, dx \, dy = 2 \int_{-1}^{1} \sqrt{1 - u^2} \cdot f(u) \, du
\]
Problem 5. (20 points) Integrate $f(x, y, z) = y$ over the region bounded by the plane $x + y + z = 2$, the cylinder $x^2 + z^2 = 1$ and $y = 0$. 